Control of the Lamb shift by a driving field

Shuai Yang,[1,*] Hang Zheng,[2,3] Ran Hong,[2] Shi-Yao Zhu,[3] and M. Suhail Zubairy[1]
[1]*Institute for Quantum Studies and Department of Physics, Texas A&M University, College Station, Texas 77843, USA*
[2]*Department of Physics, Shanghai Jiao Tong University, Shanghai 200240, People's Republic of China*
[3]*Department of Physics, Hong Kong Baptist University, Hong Kong, People's Republic of China*
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A unitary transformation approach is used to study the energy level shift of the atom coupled to both a vacuum electromagnetic field and a driving laser. The Lamb shift of the energy levels is shown to depend on the Rabi frequency and the detuning of the driving laser, which couples another pair of levels.

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I. INTRODUCTION

The Lamb shift is one of the most important quantum electrodynamics effects in atom physics and quantum optics [1]. The energy level shift in hydrogen due to the virtual photon processes, measured first by Willis Lamb, stimulated the study of the renormalized quantum field theory and confirmed the existence of the quantum vacuum. It was realized early, through the work of Bethe [2], that most of the Lamb shift can be explained within nonrelativistic quantum electrodynamics. There are a number of approaches to the calculation of the Lamb shift. One such approach is due to Feynman [3] and is beautifully reviewed by Milonni [4]. In this approach, it is argued that the presence of an atom inside a box leads to a change in the resonant frequencies from ω_k to $\omega_k/n(\omega_k)$, where $n(\omega_k)$ is the refractive index at ω_k. This leads to a change in the zero-point energy due to the presence of the atom, and the calculated change of the energy corresponds to the Lamb shift.

This motivates us to consider a situation where the refractive index $n(\omega_k)$ can be controlled by an external driving field, and hence we can coherently control the Lamb shift. Such a situation can, for example, be realized in a coherently driven system such as in electromagnetically induced transparency [5,6]. Coherent atomic effects are a hot area of research in quantum optics and have led to a number of interesting and counterintuitive phenomena, such as correlated emission laser [7,8], lasing without inversion [9,10], and suppression of atomic decay by spontaneous emission [11].

In this paper we consider a system where a coherently driven atom can lead to a coherently controlled Lamb shift. It is well known that to get the correct Lamb shift, the effect of counter-rotating terms in the interaction Hamiltonian between the atom and the electromagnetic (em) vacuum field must be included. In the regular approach to dealing with the quantum interference phenomenon, rotating wave approximation (RWA) is often made. Recently, a unitary transformation approach has been proposed to solve for the influence of the counter-rotating terms on the dynamic evolution of the atom in the short time limit [12,13]. In this paper we apply this method to a laser-driven atomic system and show how the Lamb shift can be affected by the quantum interference between the two-photon channel of the original energy levels and the new channels opened by the pumping laser. This sheds light on the feasibility of a coherently controlled Lamb shift by an extra driving laser.

II. LEVEL SHIFTS IN A COHERENTLY DRIVEN ATOM

We consider a mutilevel atom interacting with the em vacuum field as well as an extra laser field. As shown in Fig. 1, levels $|b\rangle$ and $|c\rangle$ are coupled via a coherent driving field and we are mainly interested in the Lamb shift of level $|a\rangle$. The two electric dipole allowed transitions within these three levels are $a \leftrightarrow b$ and $c \leftrightarrow b$, but the transition between $|a\rangle$ and $|c\rangle$ is considered to be dipole forbidden due to the selection rules. We suppose here that $E_b < E_c$, and the transition matrices $\mathbf{p}_{ab}$ and $\mathbf{p}_{bc}$ are perpendicular to each other. This is not difficult to choose in real system. For example, in the hydrogen atom, if we label the energy levels (n,l,m), with n being the principal quantum number, l being the orbital quantum number, and m being the magnetic quantum number, we can select $|a\rangle$, $|b\rangle$, and $|c\rangle$ to be $(2,1,1)$, $(1,0,0)$ and $(2,1,-1)$ states, respectively. The laser field $\mathbf{A}_L(\mathbf{r},t) = \mathbf{A}_D(\mathbf{r})\exp(-i\omega_D t)$ is chosen to be parallel to $\mathbf{p}_{bc}$ and is almost resonant with the $|b\rangle$ and $|c\rangle$ transition, but with large detuning with respect to other levels. So it is reasonable to suppose that the laser couples only levels $|b\rangle$ and $|c\rangle$. With the Rabi frequency associated with the driving field defined as $\Omega = \mathbf{A}_D(\mathbf{r})\cdot\mathbf{p}_{bc}$, the total Hamiltonian of the atom, the vacuum field, and the driving field is as follows:

$$\begin{aligned} H = {} & E_a|a\rangle\langle a| + E_b|b\rangle\langle b| + E_c|c\rangle\langle c| \\ & + \sum_{i\neq a,b,c} E_i|i\rangle\langle i| + \sum_k \omega_k b_k^\dagger b_k \\ & + \Omega[\exp(i\omega_D t)|b\rangle\langle c| + |c\rangle\langle b|\exp(-i\omega_D t)] \\ & + \sum_k g_{k,cb}(b_k^\dagger + b_k)(|b\rangle\langle c| + |c\rangle\langle b|) \\ & + \sum_k g_{k,ab}(b_k^\dagger + b_k)(|a\rangle\langle b| + |b\rangle\langle a|) \\ & + \sum_{n=a,b,c}\sum_{i\neq a,b,c}\sum_k g_{k,ni}(b_k^\dagger + b_k)(|n\rangle\langle i| + |i\rangle\langle n|) \\ & + \sum_{i\neq a,b,c}\sum_{j\neq a,b,c}\sum_k g_{k,ij}(b_k^\dagger + b_k)(|i\rangle\langle j| + |j\rangle\langle i|). \end{aligned} \quad (1)$$

Here we set $\hbar = 1$, E_l is the energy for level $|l\rangle$, and $b_k^\dagger$ (b_k) is the creation (annihilation) operator of the em mode with frequency ω_k (k including the polarization). The

*yangshuai@physics.tamu.edu

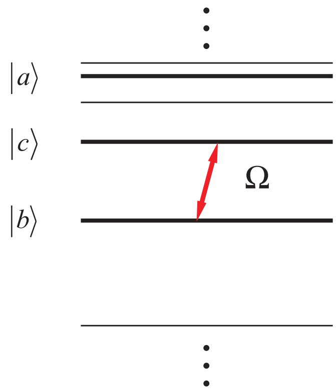

FIG. 1. (Color online) The atom configuration.

coefficient $g_{k,ij} = -e\sqrt{1/2\epsilon_0\omega_k V}\mathbf{e}_k \cdot \mathbf{p}_{ij}/m$ is the coupling constant between the atom and the vacuum field, with $\mathbf{e}_k$ being the polarization vector and $\mathbf{p}_{ij}$ being the transition matrix element of the momentum operator between level i and level j. Note that we have made a RWA for the driving-field-induced coupling with $|b\rangle$ and $|c\rangle$.

In the Hamiltonian (1) we have divided all the energy levels into two groups: the three levels $\{|a\rangle,|b\rangle,|c\rangle\}$ and other levels, labeled $|i\rangle$. Since the coupling between level $|b\rangle$ and level $|c\rangle$ is dominated by the strong coherent field and the transition $a \leftrightarrow c$ is forbidden, we can remove the terms in the fourth line (the interaction between $|b\rangle$ and $|c\rangle$ coupled to the vacuum field) and part of the sixth line [the coupling between $|b\rangle(|c\rangle)$ and the other levels]. In addition, as we are interested in the Lamb shift of state $|a\rangle$, all terms in the last line (the coupling within other levels) can also been neglected. We also make a unitary transformation, $U_0 = |a\rangle\langle a| + |b\rangle\langle b| + \exp(-i\omega_D t)|c\rangle\langle c|$, with $\omega_D = \omega_{cb} - \Delta$, where $\omega_{cb} = E_c - E_b$ and Δ is the detuning. The resulting Hamiltonian is

$$\begin{aligned}H_{\text{eff}} &= E_a|a\rangle\langle a| + E_b|b\rangle\langle b| + (\Delta + E_b)|c\rangle\langle c| \\ &\quad + \Omega(|b\rangle\langle c| + |c\rangle\langle b|) + \sum_{i\neq a,b,c} E_i|i\rangle\langle i| + \sum_k \omega_k b_k^\dagger b_k \\ &\quad + \sum_k g_{k,ab}(b_k^\dagger + b_k)(|a\rangle\langle b| + |b\rangle\langle a|) \\ &\quad + \sum_{i\neq a,b,c}\sum_k g_{k,ai}(b_k^\dagger + b_k)(|a\rangle\langle i| + |i\rangle\langle a|). \qquad (2)\end{aligned}$$

The analysis can be considerably simplified if we diagonalize the subspace corresponding to states $|b\rangle$ and $|c\rangle$. This yields the dressed states $|b'\rangle$ and $|c'\rangle$, with energies and the corresponding states given by

$$E_{b'} = E_b + \frac{1}{2}(\Delta - \sqrt{\Delta^2 + 4\Omega^2}), \qquad (3)$$

$$E_{c'} = E_b + \frac{1}{2}(\Delta + \sqrt{\Delta^2 + 4\Omega^2}), \qquad (4)$$

$$|b'\rangle = \begin{pmatrix} \cos\theta \\ -\sin\theta \end{pmatrix}, \quad |c'\rangle = \begin{pmatrix} \sin\theta \\ \cos\theta \end{pmatrix}, \qquad (5)$$

where

$$\cos\theta = \sqrt{\frac{(\Delta + \sqrt{\Delta^2 + 4\Omega^2})^2}{(\Delta + \sqrt{\Delta^2 + 4\Omega^2})^2 + 4\Omega^2}}. \qquad (6)$$

In terms of the dressed states, the Hamiltonian (2) can be rewritten as

$$H_{\text{eff}} = H_0' + H_1', \qquad (7)$$

$$\begin{aligned}H_0' &= E_a|a\rangle\langle a| + E_{b'}|b'\rangle\langle b'| + E_{c'}|c'\rangle\langle c'| \\ &\quad + \sum_{i\neq a,b',c'} E_i|i\rangle\langle i| + \sum_k \omega_k b_k^\dagger b_k, \qquad (8)\end{aligned}$$

$$\begin{aligned}H_1' &= \sum_k g_{k,ab'}(b_k^\dagger + b_k)(|a\rangle\langle b'| + |b'\rangle\langle a|) \\ &\quad + \sum_k g_{k,ac'}(b_k^\dagger + b_k)(|a\rangle\langle c'| + |c'\rangle\langle a|) \\ &\quad + \sum_{i\neq a,b',c'}\sum_k g_{k,ai}(b_k^\dagger + b_k)(|a\rangle\langle i| + |i\rangle\langle a|), \qquad (9)\end{aligned}$$

where

$$g_{k,ab'} = g_{k,ab}\cos\theta, \quad g_{k,ac'} = g_{k,ab}\sin\theta. \qquad (10)$$

As the interaction Hamiltonian H_1' contains the counter-rotating terms, we follow the unitary transformation approach presented in [12] and [13]. This approach allows us to make the unitary transformation so that the interaction part has the same form as under the RWA. First, we carry out a unitary transformation on H_{eff}; that is, $H'' = \exp(iS)H_{\text{eff}}\exp(-iS)$, with

$$S = \sum_\beta\sum_k \frac{g_{k,a\beta}\xi_{k,a\beta}}{i\omega_k}(b_k^\dagger - b_k)(|a\rangle\langle\beta| + |\beta\rangle\langle a|). \qquad (11)$$

Here we choose the index β to describe the atomic levels in the new dressed basis $\{a,b',c',\ldots\}$, and

$$\xi_{k,a\beta} = \frac{\omega_k}{\omega_k + |E_a - E_\beta|}. \qquad (12)$$

The transformation can be done order by order, $H'' = H_0'' + H_1'' + H_2'' + O(g_k^3)$, where $O(g_k^3)$ contains terms of order g_k^3 and higher, and is neglected. The first-order terms (of order g_k), $H_1'' = H_1' + [iS,H_0']$, are given by

$$\begin{aligned}H_1'' &= \sum_{E_\beta<E_a}\sum_k \frac{2g_{k,a\beta}\xi_{k,a\beta}}{\omega_k}(E_a - E_\beta)(|\beta\rangle\langle a|b_k^\dagger + |a\rangle\langle\beta|b_k) \\ &\quad + \sum_{E_\beta>E_a}\sum_k \frac{2g_{k,a\beta}\xi_{k,a\beta}}{\omega_k}(E_\beta - E_a) \\ &\quad \times (|a\rangle\langle\beta|b_k^\dagger + |\beta\rangle\langle a|b_k). \qquad (13)\end{aligned}$$

We note that H_1'' is of the same form as that of the RWA coupling. The second-order terms are $H_2'' = [iS,H_1'] + \frac{1}{2}[[iS,[iS,H_0']]]$; that is,

$$\begin{aligned}H_2'' &= -\sum_{\beta\neq a}\sum_k \frac{g_{k,a\beta}^2}{\omega_k}\left(2\xi_{k,a\beta} - \xi_{k,a\beta}^2 - \xi_{k,a\beta}^2\frac{E_\beta - E_a}{\omega_k}\right)|a\rangle\langle a| \\ &\quad - \sum_{\beta\neq a}\sum_k \frac{g_{k,a\beta}^2}{\omega_k}\left(2\xi_{k,a\beta} - \xi_{k,a\beta}^2 - \xi_{k,a\beta}^2\frac{E_a - E_\beta}{\omega_k}\right) \\ &\quad \times |\beta\rangle\langle\beta| + V_{\text{nd}}, \qquad (14)\end{aligned}$$

here V_{nd} contains the nondiagonal terms, $|a\rangle\langle\beta|$ $(\beta \neq a)$ for the atom and $b_k^\dagger b_{k'}^\dagger$, $b_k b_{k'}$ $b_k^\dagger b_{k'}$ and $b_k b_{k'}^\dagger (k \neq k')$ for the em field. Since we are interested in the energy and the decay rate of the single level, and the contribution of these nondiagonal terms would be at least third order in g_k and can be neglected, we drop $V_{\rm nd}$ in the following calculation.

The summation $\sum_k$ can be replaced by the integral,

$$\sum_k \frac{g_{k,a\beta}^2}{\omega_k} h(\omega_k) = \frac{2\alpha \mathbf{p}_{a\beta}^2}{3\pi (mc)^2} \int_0^\infty d\omega_k h(\omega_k), \tag{15}$$

where $h(\omega_k)$ is any function of ω_k and α is the fine structure constant. Then it can be easily seen that the first two terms in H_2'' are linearly divergent in the UV limit. The divergence comes from the self-energy of the free electron due to the vacuum fluctuations, $E_{\rm se} = -\sum_k \sum_{\gamma\neq\delta} (g_{k,\gamma\delta}^2/\omega_k)|\delta\rangle\langle\delta| = -\sum_{\gamma\neq\delta}[2\alpha\omega_c/3\pi(mc)^2]\mathbf{p}_{\gamma\delta}^2|\delta\rangle\langle\delta|$ ($\omega_c \approx mc^2$ is the UV cutoff), which does not depend on the atomic level structure. The divergence can be removed by the mass renormalization with the subtraction of the self-energy $E_{\rm se}$; that is,

$$\begin{aligned} H_2'' - E_{\rm se} &= -\sum_{\beta\neq a}\sum_k \frac{g_{k,a\beta}^2}{\omega_k}\left(2\xi_{k,a\beta} - \xi_{k,a\beta}^2 - 1 - \frac{E_\beta - E_a}{\omega_k}\xi_{k,a\beta}^2\right)|a\rangle\langle a| - \sum_{\beta\neq a}\sum_k \frac{g_{k,a\beta}^2}{\omega_k}\left(2\xi_{k,a\beta} - \xi_{k,a\beta}^2 - 1 - \xi_{k,a\beta}^2\frac{E_a - E_\beta}{\omega_k}\right)|\beta\rangle\langle\beta| \\ &= \frac{2\alpha}{3\pi(mc)^2}\left[\sum_{E_\beta<E_a} 2\mathbf{p}_{a\beta}^2\omega_{a\beta} + \sum_{\beta\neq a}\mathbf{p}_{a\beta}^2(E_\beta - E_a)\ln\frac{\omega_c+\omega_{a\beta}}{\omega_{a\beta}}\right]|a\rangle\langle a| + \frac{2\alpha}{3\pi(mc)^2} \\ &\quad\times \sum_{E_\beta>E_a}\left[2\mathbf{p}_{a\beta}^2\omega_{a\beta} + \mathbf{p}_{a\beta}^2(E_a - E_\beta)\ln\frac{\omega_c+\omega_{a\beta}}{\omega_{a\beta}}\right]|\beta\rangle\langle\beta| + \frac{2\alpha}{3\pi(mc)^2}\sum_{E_\beta<E_a}\mathbf{p}_{a\beta}^2(E_a - E_\beta)\ln\frac{\omega_c+\omega_{a\beta}}{\omega_{a\beta}}|\beta\rangle\langle\beta|, \end{aligned} \tag{16}$$

where $\omega_{a\beta} = |E_\beta - E_a| > 0$ is the transition frequency between level $|a\rangle$ and level $|\beta\rangle$. The transformed Hamiltonian can be written as $H_T = H_0'' + H_1''$, where $H_0'' = H_0' + H_2'' - E_{\rm se}$ is the unperturbed part; that is,

$$H_0'' = \sum_{\beta\in\{a,b',c',\ldots\}} E_\beta''|\beta\rangle\langle\beta| + \sum_k \omega_k b_k^\dagger b_k, \tag{17}$$

with

$$E_a'' = E_a + \frac{2\alpha}{3\pi(mc)^2} \times \left[\sum_{E_\beta<E_a} 2\mathbf{p}_{a\beta}^2\omega_{a\beta} + \sum_{\beta\neq a}\mathbf{p}_{a\beta}^2(E_\beta - E_a)\ln\frac{\omega_c+\omega_{a\beta}}{\omega_{a\beta}}\right], \tag{18}$$

$$E_\beta'' = E_\beta + \frac{2\alpha}{3\pi(mc)^2} \left[2\mathbf{p}_{a\beta}^2\omega_{a\beta} + \mathbf{p}_{a\beta}^2(E_a - E_\beta)\ln\frac{\omega_c+\omega_{a\beta}}{\omega_{a\beta}}\right] \quad (E_\beta > E_a), \tag{19}$$

$$E_\beta'' = E_\beta + \frac{2\alpha}{3\pi(mc)^2}\mathbf{p}_{a\beta}^2(E_a - E_\beta)\ln\frac{\omega_c+\omega_{a\beta}}{\omega_{a\beta}} \quad (E_\beta < E_a), \tag{20}$$

and H_1'' in Eq. (13) represents the perturbation. The contribution H_T is the approximately diagonalized Hamiltonian for this coupled system of the multilevel atom and the em field because the UV divergence has already been subtracted and the mass renormalization has been done.

The second-order perturbed energy of level a due to the transformed interaction Hamiltonian H_1'' is

$$\begin{aligned} \Delta E_a^{(2)} &= \sum_\beta\sum_k \frac{|\langle\beta,1_k|H_1''|a,\,{\rm vac}\rangle|^2}{E_a - E_\beta - \omega_k} \\ &= \sum_{E_\beta<E_a}\sum_k \frac{4g_{k,a\beta}^2\xi_{k,a\beta}^2\omega_{a\beta}^2}{\omega_k^2(\omega_{a\beta}-\omega_k)} \\ &\cong -\frac{2\alpha}{3\pi(mc)^2}\sum_{E_\beta<E_a} 2\mathbf{p}_{a\beta}^2\omega_{a\beta}, \end{aligned} \tag{21}$$

where $|a,\,{\rm vac}\rangle$ corresponds to the atom in level a and no photon, and the intermediate state $|\beta,1_k\rangle$ corresponds to the atom in level β and one photon in mode k. When summing over k, we simplify the result using the fact that the UV cutoff $\omega_c \approx mc^2$ is much larger than the atomic level energy and get the third line.

Now the Lamb shift of level a is given by

$$\begin{aligned} E_a^{\rm Lamb} &= E_a'' - E_a + \Delta E_a^{(2)} \\ &= \frac{2\alpha}{3\pi(mc)^2}\sum_{\beta\neq a}\mathbf{p}_{a\beta}^2(E_\beta - E_a)\ln\frac{\omega_c+\omega_{a\beta}}{\omega_{a\beta}} \\ &\cong \frac{2\alpha}{3\pi(mc)^2}\sum_{\beta\neq a}\mathbf{p}_{a\beta}^2(E_\beta - E_a)\ln\frac{\omega_c}{\omega_{a\beta}}. \end{aligned} \tag{22}$$

For the last step we utilize $\omega_c \gg \omega_{a\beta}$.

It is clear from comparison of Eq. (22) with the Lamb shift without the driving field [4] that the only difference between the two cases is that bases b and c are shifted to the dressed bases b' and c' when adding the driving laser. The Lamb shift is therefore changed accordingly, and the resulting change in the Lamb shift of level a due to the addition of the driving field is therefore given by

$$\begin{aligned}\Delta E_a^{\text{Lamb}} &= \frac{2\alpha}{3\pi(mc)^2}\left[\mathbf{p}_{ab'}^2(E_b'-E_a)\ln\frac{\omega_c}{|E_b'-E_a|}\right.\\&\quad\left.+\mathbf{p}_{ac'}^2(E_c'-E_a)\ln\frac{\omega_c}{|E_c'-E_a|}\right]\\&\quad-\frac{2\alpha}{3\pi(mc)^2}\mathbf{p}_{ab}^2(E_b-E_a)\ln\frac{\omega_c}{|E_b-E_a|}\\&=\chi\left\{\cos^2\theta\left[\omega_{ab}-\frac{1}{2}(\Delta-\sqrt{\Delta^2+4\Omega^2})\right]\right.\\&\quad\times\ln\frac{\omega_{ab}-\frac{1}{2}(\Delta-\sqrt{\Delta^2+4\Omega^2})}{\omega_{ab}}\\&\quad+\sin^2\theta\left[\omega_{ab}-\frac{1}{2}(\Delta+\sqrt{\Delta^2+4\Omega^2})\right]\\&\quad\left.\times\ln\frac{\omega_{ab}-\frac{1}{2}(\Delta+\sqrt{\Delta^2+4\Omega^2})}{\omega_{ab}}\right\},\end{aligned}\tag{23}$$

where $\chi=2\alpha\mathbf{p}_{ab}^2/(3\pi m^2c^2)$. In the first line, the contribution proportional to $\mathbf{p}_{ac}^2$ is missing, as the coupling between level a and level c is dipole forbidden, $\mathbf{p}_{ac}=0$. This is the main result of this paper. Since this change in the Lamb shift depends on the Rabi frequency and the detuning of the driving filed (mainly the Rabi frequency; see Fig. 2), we can coherently control the Lamb shift by changing the laser field. In particular, for those levels which initially have a zero Lamb shift (e.g., the 2P state for the hydrogen atom), we can produce a tunable Lamb shift. In the case of a resonant driving field, $\Delta=0$, we obtain $\cos^2\theta=1/2$. The additional Lamb shift by the driving field is then

$$\begin{aligned}\Delta E_a^{\text{Lamb}} &= \frac{1}{2}\chi\omega_{ab}\left[\left(1+\frac{\Omega}{\omega_{ab}}\right)\ln\left(1+\frac{\Omega}{\omega_{ab}}\right)\right.\\&\quad\left.+\left(1-\frac{\Omega}{\omega_{ab}}\right)\ln\left(1-\frac{\Omega}{\omega_{ab}}\right)\right].\end{aligned}\tag{24}$$

We note that in the preceding discussion, we do not make any assumption about the energy relation between E_a and $E_b(E_c)$, since the Lamb shift is due to the processes of emission and reabsorption of the virtual vacuum photons between level $|a\rangle$ and any other possible level as long as the dipole moment is nonzero.

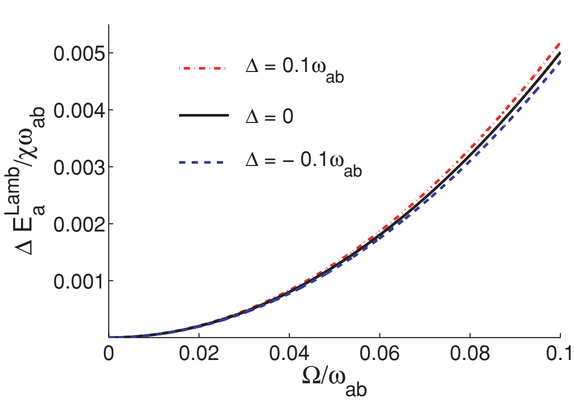

FIG. 2. (Color online) Additional Lamb shift by the driving field.

Next we suppose that level $|b\rangle$ is the ground state of the atom and $E_a>E_c'>E_b'$. We study the influence on the decay rate of the level $|a\rangle$ of the driving filed. The total Hamiltonian is $H_T=H_0''+H_1''$ and it is in the form of the RWA. There are two decay channels for the electron in level $|a\rangle$, that is, $a\to b'$ and $a\to c'$. The effective decay rate of the energy level $|a\rangle$ for a short time is [12]

$$\begin{aligned}\gamma(\tau)&=2\pi\int d\omega[G_{ab'}'(\omega)F(\omega-\omega_{ab'},\tau)\\&\quad+G_{ac'}'(\omega)F(\omega-\omega_{ac'},\tau)],\end{aligned}\tag{25}$$

where

$$G_{ai'}'(\omega)=\frac{2\alpha}{3\pi(mc)^2}\frac{4\omega_{ai'}^2}{(\omega+\omega_{ai'})^2}\mathbf{p}_{ai'}^2\omega,\tag{26}$$

$$F(\omega-\omega_{ai'},\tau)=2\frac{\sin^2\left[\frac{\omega-\omega_{ai'}}{2}\tau\right]}{\pi\tau(\omega-\omega_{ai'})^2},\tag{27}$$

with $i'=b',c'$. If there is no driving field, $\Omega=0$, we have $\cos\theta=1$. We then have $G_{ac'}'=0$ and $\omega_{ab'}=\omega_{ab}$, and the decay rate goes back to the same form as in [12]. In the long time limit, $F(\omega-\omega_{ai'},\tau)\to\delta(\omega-\omega_{ai'})$, the decay rate is given by

$$\gamma(\Delta,\Omega)=2\pi\frac{2\alpha}{3\pi(mc)^2}\mathbf{p}_{ab}^2(\cos^2\theta\omega_{ab'}+\sin^2\theta\omega_{ac'}).\tag{28}$$

If the driving field is at resonance, $\cos^2\theta=1/2$, $\omega_{ab'}=\omega_{ab}-\Omega$, $\omega_{ac'}=\omega_{ab}+\Omega$, and we obtain

$$\gamma(\Omega)=2\pi\frac{2\alpha}{3\pi(mc)^2}\mathbf{p}_{ab}^2\omega_{ab}.\tag{29}$$

In this case, the driving field has no effect on the decay rate. This coincides with the result in [14], where the RWA of the coupling between the atom and the vacuum field is made at the beginning. This result can be understood from the observation that, in the long time limit, the counter-rotating terms have no influence on the decay.

As mentioned in the Introduction, this problem is motivated by Feynman's derivation of the Lamb shift. In this interpretation, a dilute gas with N atoms per unit volume in a box of volume V is considered [4]. Since the dimension of the box determines the allowed wavelengths in the box, the frequencies would be affected by the refractive index associated with the atomic gas. Thus the presence of the atomic gas changes the zero-point field energy by

$$\begin{aligned}\Delta E&=\sum_{\mathbf{k}}\frac{1}{2}\frac{\hbar\omega_k}{n(\omega_k)}-\sum_{\mathbf{k}}\frac{1}{2}\hbar\omega_k\\&\cong-\sum_{\mathbf{k}}[n(\omega_k)-1]\frac{1}{2}\hbar\omega_k,\end{aligned}\tag{30}$$

for $n(\omega_k)\cong1$, where $n(\omega_k)$ is the refraction index of the atomic gas at ω_k. For a dilute gas of atoms in level $|a\rangle$,

$$n\cong1+\frac{4\pi N}{3\hbar}\sum_l\frac{\omega_{al}|\mathbf{d}_{al}|^2}{\omega_{al}^2-\omega_k^2},\tag{31}$$

where $\mathbf{d}_{al}$ is the $a\leftrightarrow l$ transition dipole moment. Then by subtracting from this expression the change in the zero-point energy due to free electrons with the same density, we can get an observable energy shift for the atomic level $|a\rangle$.

For the case of an atom gas with the driving field just considered, we note that the refractive index of the gas with the atom in level $|a\rangle$ changes to

$$n' \cong n + \frac{4\pi N}{3\hbar}\left(\sum_{\beta=b',c'}\frac{\omega_{a\beta}|\mathbf{d}_{a\beta}|^2}{\omega_{a\beta}^2-\omega_k^2} - \frac{\omega_{ab}|\mathbf{d}_{ab}|^2}{\omega_{ab}^2-\omega_k^2}\right). \quad (32)$$

Thus the change in the zero-point energy due to the presence of the driving field is given by

$$\Delta E_a = -\sum_{\mathbf{k}}\frac{\hbar\omega_k}{2}\frac{4\pi N}{3\hbar}\left(\sum_{\beta=b',c'}\frac{\omega_{a\beta}|\mathbf{d}_{a\beta}|^2}{\omega_{a\beta}^2-\omega_k^2} - \frac{\omega_{ab}|\mathbf{d}_{ab}|^2}{\omega_{ab}^2-\omega_k^2}\right). \quad (33)$$

As the box contains only one atom, we have $NV = 1$. After summing over $\mathbf{k}$ and recalling that $|\mathbf{p}_{al}|^2 = m^2\omega_{al}^2|\mathbf{x}_{al}|^2 = (m^2\omega_{al}^2/e^2)|\mathbf{d}_{al}|^2$, we obtain the same Lamb shift as derived previously.

III. CONCLUSION

In this work, we have studied the effect of the driving laser on the Lamb shift. First we change the picture of the system from a bare atom and laser field to a dressed state. Then a unitary transformation is made on the original Hamiltonian, which is then transformed into the form of a RWA. We can directly show that the Lamb shift depends on the Rabi frequency and the detuning of the driving field. This relation provides a way to control the Lamb shift coherently.

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