Interference between driving and dissipation in the spin-boson model: Effect of counter-rotating terms

Chunjian Zhao and Hang Zheng
Key Laboratory of Artificial Structures and Quantum Control (Ministry of Education), Department of Physics, Shanghai Jiao Tong University, Shanghai 200240, China
(Received 21 July 2010; published 29 October 2010)

The quantum dynamics of the dissipative two-level system in the Ohmic bath with Rabi driving is studied by means of a modified Bloch-Redfield approach based on a unitary transformation. The main purpose is to understand the effect of counter-rotating terms of the system-environment interaction on the dynamics of the two-level system and, in particular, if it is possible to control the effective system-environment coupling by their effect on the interference between driving and dissipation. By calculating the nonequilibrium correlation $P(t)$ for a finite driving it is shown that the counter-rotating terms of the system-environment interaction play an important role in the dynamic evolution with a longer coherent time and a weaker amplitude of the long-time driven oscillation.

DOI: 10.1103/PhysRevA.82.043844 PACS number(s): 42.50.Ct, 05.30.Jp

I. INTRODUCTION

The physics of a driven two-level system coupled to a dissipative bosonic environment [driven spin-boson model (DSBM)] has attracted considerable attention in recent years because it provides a universal model for numerous physical and chemical processes [1–3]. The Hamiltonian of DSBM reads (we set $\hbar = 1$)

$$H(t) = -\frac{1}{2}\Delta\sigma_x + \epsilon(t)\sigma_z + \sum_k \omega_k b_k^\dagger b_k + \frac{1}{2}\sum_k g_k(b_k^\dagger + b_k)\sigma_z. \tag{1}$$

$b_k^\dagger$ (b_k) is the creation (annihilation) operator of the boson mode with frequency ω_k, and σ_x and σ_z are Pauli matrices to describe the two-level system. $\epsilon(t)$ is the time-dependent driving force, Δ the bare tunneling, and g_k the coupling between spin and environment.

The essential physics contained in DSBM is the competition between the coherent quantum dynamics of the two-level system [the driving two-level system described by the first two term of Eq. (1)] and the dissipative effect of the environment which tends to make the dynamics decoherent. The main theoretical interest is to understand how the interplay between the driving and the dissipation influence the decoherence and, in particular, if it is possible to control the effective system-environment coupling by using the interference between the driving and the dissipative effect of the environment [4,5]. The dynamical evolution of primary interest is defined as

$$P(t) = \mathrm{Tr}_S\{\mathrm{Tr}_B[\rho_{SB}(t)\sigma_z]\}, \tag{2}$$

where $\rho_{SB}(t)$ is the density operator for the Hamiltonian $H(t)$ satisfying the equation,

$$i\frac{\partial}{\partial t}\rho_{SB}(t) = [H(t), \rho_{SB}(t)], \tag{3}$$

and the subscript SB indicates that it is a density operator for the coupled two-level system (S) and bath (B). Besides, the effect of the bosonic environment is characterized by a spectral density $J(\omega) = \sum_k g_k^2\delta(\omega - \omega_k) = 2\alpha\omega\theta(\omega_c - \omega)$ with the dimensionless coupling strength α and the hard upper cutoff ω_c [$\theta(\omega_c - \omega)$ is the usual step function]. Here $J(\omega)$ is the spectrum of the Ohmic bath [1,2].

The DSBM has been studied by several approximate analytic approaches. The first one is the traditional optical Bloch equations approach [6–8] where the constant nondiagonal and diagonal relaxation rates were used in the Bloch equations. The second one is the approach of the Bloch-Redfield master equation [4], where the Born-Markov approximation was used for decoupling the master equation. Besides, the polaron transformation is used for the DSBM, which is an expansion over the tunneling Δ until the second order Δ^2 [4] and is a good approximation for small tunneling Δ and strong coupling (large α). Dekker [9] proved that the polaron transformation is equivalent to the real-time path-integral methods such as the noninteracting blip approximation (NIBA) [10]. Hartmann *et al.* have illuminated the advantages and disadvantages of the Bloch-Redfield theory versus the path-integral approach [11]. In general, to obtain a solution for DSBM even numerically is a nontrivial task [12–14], as the bosonic bath is of infinite degrees of freedom.

It is well known that, in the Bloch equations approach and in that of the Bloch-Redfield master equation, the counter-rotating terms of the system-environment interaction contribute nothing to the relaxation rates because they are higher frequency terms under Born-Markov approximation and their long-time averages are zero [15]. The approximation omitting the contribution of counter-rotating terms is usually referred to as the rotating-wave approximation (RWA), which may be a good approximation for weak driving $\epsilon(t)/\Delta \ll 1$ because there is only one characteristic energy Δ for the quantum system. Now, the question is: When the driving is not weak $\epsilon(t)/\Delta \leqslant 1$, does the RWA still work?

For the finite driving, the counter-rotating terms may contribute to the interference between the driving and the dissipation as $\epsilon(t)$ appears as another characteristic energy for the quantum system. Previously, we developed an analytic approach to take into account the effect of the counter-rotating terms by unitary transformation and then diagonalizing the transformed Hamiltonian [16]. In [17] the approach was applied to the DSBM (1) and the dynamical evolution $P(t)$ was calculated. But that diagonalization approach may not be suitable for the case of moderate driving $\epsilon(t)/\Delta \leqslant 1$ because when doing the diagonalization only the following bare states are taken into account: $|s_1\rangle|\{0_k\}\rangle$, $|s_2\rangle|\{0_k\}\rangle$, and $|s_1\rangle|\{1_k\}\rangle$. Here $|s_{1(2)}\rangle$ is the eigenstate of σ_x: $\sigma_x|s_{1(2)}\rangle = +(-)|s_{1(2)}\rangle$, and $|\{0_k\}\rangle$ the vacuum state of the bath but $|\{1_k\}\rangle$ the excited state of the bath with only one excitation at mode k. When the driving force is not weak, the other excited states, such as $|s_2\rangle|\{1_k\}\rangle$, must be taken into account because the driving $\epsilon(t)$ can pump between $|s_1\rangle|\{1_k\}\rangle$ and $|s_2\rangle|\{1_k\}\rangle$.

When taking into account more excited states, the diagonalization approach becomes quite complicated but the master equation approach for the density operator can work. In this paper, we propose a modified Bloch-Redfield approach for solving the master equation of density operator of the DSBM, which is based on the same unitary transformation as that of Refs. [16,17]. By means of the transformation, the effect of the counter-rotating terms of the system-environment interaction has been taken into account even if we make the Born-Markov approximation after the transformation. Because the master equation approach has been used, we will show that the pumping by $\epsilon(t)$ between $|s_1\rangle|\{1_k\}\rangle$ and $|s_2\rangle|\{1_k\}\rangle$ leads to interference between the driving and the dissipative effect of the environment. Our calculation will lead to different results from those of Ref. [17] and one can see that the counter-rotating terms play an important role in the quantum dynamics of the DSBM.

II. UNITARY TRANSFORMATION

We present a treatment based on the unitary transformation approach. A unitary transformation [16,17] is applied to H, $H' = \exp(S)H\exp(-S)$, and the purpose of the transformation is to take into account the correlation between the spin and bosons, where

$$S = \sum_k \frac{g_k}{2\omega_k}\xi_k(b_k^\dagger - b_k)\sigma_z. \tag{4}$$

Here, we introduce in S a k-dependent function ξ_k; its form will be determined later.

The transformation can be done to the end and the result is

$$H'(t) = H'_0(t) + H'_1 + H'_2, \tag{5}$$

$$H'_0(t) = -\frac{1}{2}\eta\Delta\sigma_x + \epsilon(t)\sigma_z + \sum_k \omega_k b_k^\dagger b_k - \sum_k \frac{g_k^2}{4\omega_k}\xi_k(2-\xi_k), \tag{6}$$

$$H'_1 = \frac{1}{2}\sum_k g_k(1-\xi_k)(b_k^\dagger + b_k)\sigma_z - \frac{1}{2}\eta\Delta i\sigma_y \sum_k \frac{g_k}{\omega_k}\xi_k(b_k^\dagger - b_k), \tag{7}$$

$$H'_2 = -\frac{1}{2}\Delta\sigma_x\left(\cosh\left\{\sum_k \frac{g_k}{\omega_k}\xi_k(b_k^\dagger - b_k)\right\} - \eta\right) - \frac{1}{2}\Delta i\sigma_y \times\left(\sinh\left\{\sum_k \frac{g_k}{\omega_k}\xi_k(b_k^\dagger - b_k)\right\} - \eta\sum_k \frac{g_k}{\omega_k}\xi_k(b_k^\dagger - b_k)\right), \tag{8}$$

where

$$\eta = \left\langle \{0_k\}\left|\cosh\left\{\sum_k \frac{g_k}{\omega_k}\xi_k(b_k^\dagger - b_k)\right\}\right|\{0_k\}\right\rangle = \exp\left[-\sum_k \frac{g_k^2}{2\omega_k^2}\xi_k^2\right] \tag{9}$$

is an average over the vacuum state of the bath.

Obviously, $H'_0(t)$ can be solved exactly because the spin and bosons are decoupled, and its first two terms can be rewritten as

$$-\tfrac{1}{2}\eta\Delta(|s_1\rangle\langle s_1| - |s_2\rangle\langle s_2|) + \epsilon(t)(|s_2\rangle\langle s_1| + |s_1\rangle\langle s_2|).$$

If the functional form of ξ_k is determined as

$$\xi_k = \frac{\omega_k}{\omega_k + \eta\Delta}, \tag{10}$$

then we have

$$H'_1 = \sum_k V_k[b_k^\dagger|s_1\rangle\langle s_2| + b_k|s_2\rangle\langle s_1|], \tag{11}$$

where $V_k = \eta\Delta g_k\xi_k/\omega_k$. Note that H'_1 contains only the rotating wave term, that is, because of the unitary transformation the counter-rotating terms disappear. However, the renormalized coupling $V_k = \eta\Delta g_k/(\omega_k + \eta\Delta)$ already takes into account the effect of the counter-rotating terms and $V_k \to g_k/2$ when the bosonic frequency $\omega_k \to \eta\Delta$. We note that $g_k/2$ is the bare coupling in the original Hamiltonian (1).

The transformed Hamiltonian $H'(t) = H'_0(t) + H'_1 + H'_2$ is equivalent to the original $H(t)$ and there is no approximation until this point. In the following, the transformed Hamiltonian is approximated as $H'(t) \approx H'_0(t) + H'_1$, since $\langle\{0_k\}|H'_2|\{0_k\}\rangle = 0$ [this is the reason for determining the value of η in Eq. (9)]. It can be checked that, after the transformation, η contains the contribution from zero-boson transition and the terms of single-boson transition are included in H'_1. The terms contained in $\tilde{H}_2$ are related to the double- and multiboson nondiagonal transition (like $b_k b_{k'}$ and $b_k^\dagger b_{k'}^\dagger$) and their contributions to the physical quantities are $O(g_k^4)$. In the zero-temperature case the contribution from these multiboson nondiagonal transitions may be dropped safely.

III. DENSITY OPERATOR AND MASTER EQUATION

The density operator in the Schroedinger representation is $\rho_{SB}(t)$ with Hamiltonian $H(t)$. For transformed Hamiltonian $H'(t) \approx H'_0(t) + H'_1$ the density operator is

$$\rho'_{SB}(t) = e^S\rho_{SB}(t)e^{-S} = U'(t)\rho'_{SB}(0)U'^\dagger(t), \tag{12}$$

where $\rho'_{SB}(0)$ is the initial density operator for $H'(t)$ and the propagator $U'(t)$ satisfies the Schroedinger equation [4,18],

$$i\frac{\partial}{\partial t}U'(t) = H'(t)U'(t). \tag{13}$$

In this work, a harmonic driving $\epsilon(t) = \Omega\cos(\omega_L t)$ is used as the driving force, where Ω is the Rabi frequency and ω_L the carrier frequency. Generally speaking, $\cos(\omega_L t) \neq \cos(\omega_L t')$ when $t \neq t'$, thus $H'_0(t)$ and $H'_0(t')$ do not commute with each other and the master equation for $\rho'_{SB}(t)$ is quite difficult to be solved. As our main purpose is to see how the driving field modifies the relaxation and damping rate, in this work we treat the resonant case of the driving field with $\omega_L = \eta\Delta$. Because of the resonant case the propagator may be transformed to the rotating frame, $\tilde{U}(t) = X(t)U'(t)$,

$$X(t) = \exp\left\{i\eta\Delta t\left[(|s_2\rangle\langle s_2| - |s_1\rangle\langle s_1|)/2 + \sum_k b_k^\dagger b_k\right]\right\}, \quad (14)$$

which satisfies the transformed Schroedinger equation,

$$i\frac{\partial}{\partial t}\tilde{U}(t) = \left[\sum_k(\omega_k - \eta\Delta)b_k^\dagger b_k + \frac{\Omega}{2}(|s_2\rangle\langle s_1| + |s_1\rangle\langle s_2|) + \frac{\Omega}{2}(|s_2\rangle\langle s_1|e^{i(\omega_L+\eta\Delta)t} + |s_1\rangle\langle s_2|e^{-i(\omega_L+\eta\Delta)t}) + H_1'\right]\tilde{U}(t). \quad (15)$$

The two high-frequency terms, $\exp[i(\omega_L + \eta\Delta)t]$ and $\exp[-i(\omega_L + \eta\Delta)t]$, will be omitted. Then,

$$\tilde{H}_0 = \tilde{H}_{0B} + \tilde{H}_{0S} = \sum_k(\omega_k - \eta\Delta)b_k^\dagger b_k + \frac{\Omega}{2}(|s_2\rangle\langle s_1| + |s_1\rangle\langle s_2|) \quad (16)$$

is time independent, where $\tilde{H}_{0B}$ is the bath term and $\tilde{H}_{0S} = \Omega(|s_2\rangle\langle s_1| + |s_1\rangle\langle s_2|)/2$. The density operator in the rotating frame is $\tilde{\rho}_{SB}(t) = \tilde{U}(t)\rho'_{SB}(0)\tilde{U}^\dagger(t)$. The reduced density operator is $\tilde{\rho}_S(t) = \mathrm{Tr}_B\tilde{\rho}_{SB}(t)$ which satisfies the following master equation [4,18],

$$\frac{d}{dt}\tilde{\rho}_S(t) = -i[\tilde{H}_{0S},\tilde{\rho}_S(t)] - \int_0^t dt'\mathrm{Tr}_B\left[H_1'(t),e^{i\tilde{H}_{0S}(t'-t)}[H_1'(t'),\tilde{\rho}_S(t)\rho_B]e^{-i\tilde{H}_{0S}(t'-t)}\right], \quad (17)$$

where we have used the Born-Markov approximation and

$$H_1'(t) = \sum_k V_k[b_k^\dagger|s_1\rangle\langle s_2|e^{i(\omega_k-\eta\Delta)t} + b_k|s_2\rangle\langle s_1|e^{-i(\omega_k-\eta\Delta)t}]. \quad (18)$$

The details of deriving the master equation are listed in the appendix. In the long time limit, the master equation (A8) can be rewritten for four elements of the density operator,

$$\tilde{\rho}_S(t) = \begin{pmatrix}\tilde{\rho}_{11}(t) & \tilde{\rho}_{12}(t)\\ \tilde{\rho}_{21}(t) & \tilde{\rho}_{22}(t)\end{pmatrix},$$

$$\frac{d}{dt}\tilde{\rho}_{11}(t) = -i\frac{\Omega}{2}[\tilde{\rho}_{21}(t) - \tilde{\rho}_{12}(t)] - \left[\gamma(\eta\Delta) + \frac{1}{2}\gamma(\eta\Delta - \Omega) + \frac{1}{2}\gamma(\eta\Delta + \Omega)\right]\tilde{\rho}_{11}(t) - \frac{1}{4}[\gamma(\eta\Delta + \Omega) - \gamma(\eta\Delta - \Omega)][\tilde{\rho}_{21}(t) + \tilde{\rho}_{12}(t)], \quad (19)$$

$$\frac{d}{dt}[\tilde{\rho}_{21}(t) - \tilde{\rho}_{12}(t)] = i\Omega[\tilde{\rho}_{22}(t) - \tilde{\rho}_{11}(t)] - \frac{1}{2}[\gamma(\eta\Delta - \Omega) + \gamma(\eta\Delta + \Omega)][\tilde{\rho}_{21}(t) - \tilde{\rho}_{12}(t)], \quad (20)$$

$$\frac{d}{dt}[\tilde{\rho}_{21}(t) + \tilde{\rho}_{12}(t)] = -[\gamma(\eta\Delta + \Omega) - \gamma(\eta\Delta - \Omega)]\tilde{\rho}_{11}(t) - \frac{1}{2}[\gamma(\eta\Delta - \Omega) + \gamma(\eta\Delta + \Omega)][\tilde{\rho}_{21}(t) + \tilde{\rho}_{12}(t)], \quad (21)$$

$$\frac{d}{dt}\tilde{\rho}_{22}(t) = -\frac{d}{dt}\tilde{\rho}_{11}(t). \quad (22)$$

Here, $\gamma(\omega)$ is the following time integral,

$$\gamma(\omega) = \sum_k V_k^2\int_0^t dt'\exp[-i(\omega_k - \omega)(t - t')] = \sum_k\left(\frac{\eta\Delta g_k}{\omega_k + \eta\Delta}\right)^2\frac{1 - \exp[-i(\omega_k - \omega)t]}{i(\omega_k - \omega)}. \quad (23)$$

When $\omega t \gg 1$, we have

$$\gamma(\omega) = \pi\sum_k\left(\frac{\eta\Delta g_k}{\omega_k + \eta\Delta}\right)^2\delta(\omega_k - \omega) = \pi J(\omega)\left(\frac{\eta\Delta}{\omega + \eta\Delta}\right)^2 \quad (24)$$

where we have omitted the imaginary part of $\gamma(\omega)$ since it is a small level shift which is not important. We note that the renormalization function $[2\eta\Delta/(\omega + \eta\Delta)]^2$ comes from our unitary transformation, which takes into account the effect of the counter-rotating terms but the transformed interaction H_1' contains the rotating-wave terms only [see Eq. (18)]. If we start from $H(t) = H_0(t) + H_1$ in Eq. (1), do not make the unitary transformation (3) but follow the same procedure of the Born-Markov approximation from (12) to (18), we may arrive at the same differential Eqs. (19)–(22) [in this way the counter-rotating terms in (1) contribute nothing in the second order of $O(g_k^2)$] but with nonrenormalized

$$\gamma_0(\omega) = \pi\sum_k\frac{g_k^2}{4}\delta(\omega_k - \omega) = \pi J(\omega)/4, \quad (25)$$

instead of the renormalized $\gamma(\omega)$ in Eq. (24). Note that at resonant point $\omega = \omega_L = \eta\Delta$, we have $\gamma(\eta\Delta) = \gamma_0(\Delta) = \gamma_0$ with $\gamma_0 = 0.5\pi\alpha\omega_L$.

The solution of Eqs. (19)–(22) with nonrenormalized $\gamma_0(\Delta)$ and $\gamma_0(\Delta \pm \Omega)$ will be denoted as the RWA solution. We will show that when the ratio Ω/Δ is not very small, the counter-rotating terms have significant effect on the dynamical evolution.

IV. DYNAMICAL EVOLUTION

One of our purposes is to calculate the nonequilibrium correlation $P(t)$ [Eq. (2)]. Because of the unitary transforms in Eq. (12), it can be calculated as

$$\begin{aligned}P(t) &= \mathrm{Tr}_S(\mathrm{Tr}_B(e^S\rho_{SB}(t)e^{-S}\sigma_z))\\ &= \mathrm{Tr}_S(\mathrm{Tr}_B(U'(t)\rho'_{SB}(0)U'^\dagger(t)\sigma_z))\\ &= \mathrm{Tr}_S(\mathrm{Tr}_B(\tilde{U}(t)\rho'_{SB}(0)\tilde{U}^\dagger(t)X(t)\sigma_zX^\dagger(t)))\\ &= \mathrm{Tr}_S(\tilde{\rho}_S(t)(|s_2\rangle\langle s_1|e^{i\omega_Lt} + |s_1\rangle\langle s_2|e^{-i\omega_Lt}))\\ &= \tilde{\rho}_{12}(t)e^{-i\omega_Lt} + \tilde{\rho}_{21}(t)e^{i\omega_Lt}\\ &= [\tilde{\rho}_{21}(t) + \tilde{\rho}_{12}(t)]\cos(\omega_Lt) + i[\tilde{\rho}_{21}(t) - \tilde{\rho}_{12}(t)]\sin(\omega_Lt).\end{aligned} \quad (26)$$

$\tilde{\rho}_{12}(t)$ and $\tilde{\rho}_{21}(t)$ are solutions of Eqs. (19)–(22). When solving Eqs. (19)–(22) we use the initial density operator $\rho_S(0) = \frac{1}{2}\left(\begin{smallmatrix}1 & 1\\ 1 & 1\end{smallmatrix}\right)$, which leads to the following solutions:

$$\begin{aligned}
&\tilde{\rho}_{21}(t)+\tilde{\rho}_{12}(t)\\
&\quad= -\frac{\Gamma_-\Omega^2}{\Gamma_+\left(\Omega_r^2+\Gamma^2\right)} + \left[1+\frac{\Gamma_-}{\Gamma_+}\frac{\Omega^2+\Gamma_+\Gamma_-/4}{\Omega^2-\Gamma_-^2/4}\right]e^{-\Gamma_+ t/2}\\
&\qquad - \frac{\Gamma_-}{\Omega^2-\Gamma_-^2/4}\left[\frac{\Omega^2(\gamma_0+\Gamma_+/2)/2}{\Omega_r^2+\Gamma^2}+\frac{\Gamma_-}{4}\right]\cos(\Omega_r t)e^{-\Gamma t}\\
&\qquad - \frac{\Gamma_-}{2\Omega_r}\left\{1+\frac{1}{\Omega^2-\Gamma_-^2/4}\left[\frac{\gamma_0\Gamma_-}{4}\right.\right.\\
&\qquad \left.\left. + \frac{\Omega^2\left(\Gamma^2-\Omega_r^2-\frac{1}{2}\Gamma_+\Gamma\right)}{\Omega_r^2+\Gamma^2}\right]\right\}\sin(\Omega_r t)e^{-\Gamma t},
\end{aligned} \tag{27}$$

$$\begin{aligned}
&i[\tilde{\rho}_{21}(t)-\tilde{\rho}_{12}(t)]\\
&\quad= -\frac{2\Omega}{\Gamma_+}\left[1-\frac{\Omega^2}{\Omega_r^2+\Gamma^2}\right] + \frac{2\Omega}{\Gamma_+}\left[1-\frac{\Omega^2+\Gamma_+\Gamma_-/4}{\Omega^2-\Gamma_-^2/4}\right]\\
&\qquad \times e^{-\Gamma_+ t/2} + \frac{\Omega}{\Omega^2-\Gamma_-^2/4}\left[\frac{\Omega^2(\gamma_0+\Gamma_+/2)}{\Omega_r^2+\Gamma^2}+\frac{\Gamma_-}{2}\right]\\
&\qquad \times \cos(\Omega_r t)e^{-\Gamma t} + \frac{\Omega}{\Omega_r}\left\{1+\frac{1}{\Omega^2-\Gamma_-^2/4}\right.\\
&\qquad \left. \times\left[\frac{\gamma_0\Gamma_-}{4}+\frac{\Omega^2\left(\Gamma^2-\Omega_r^2-\frac{1}{2}\Gamma_+\Gamma\right)}{\Omega_r^2+\Gamma^2}\right]\right\}\sin(\Omega_r t)e^{-\Gamma t},
\end{aligned} \tag{28}$$

where

$$\Omega_r = \sqrt{\Omega^2-\left(\gamma_0^2+\Gamma_-^2\right)/4}, \tag{29}$$

$$\Gamma_\pm = \gamma(\eta\Delta+\Omega) \pm \gamma(\eta\Delta-\Omega), \tag{30}$$

and $\Gamma = [\gamma_0+\Gamma_+]/2$. One can see that there are two decay rates for the dynamical evolution $P(t)$: one is $\Gamma_+/2$ for the terms with frequency ω_L and the other is $\Gamma = (\gamma_0+\Gamma_+)/2$ for the terms with frequency $\omega_L \pm \Omega_r$. We note that, for Ohmic bath $J(\omega) = 2\alpha\omega\theta(\omega_c-\omega)$ and when $\Omega > 0$, the decay rates of RWA solution are larger than the rates of our solution:

$$\Gamma_{+RWA} = \gamma_0(\Delta+\Omega)+\gamma_0(\Delta-\Omega) = 2\gamma_0, \tag{31}$$

$$\begin{aligned}
\Gamma_+ &= \gamma(\eta\Delta+\Omega)+\gamma(\eta\Delta-\Omega)\\
&= 2\gamma_0\left(1-\frac{3}{4}\frac{\Omega^2}{\omega_L^2}\right)\Big/\left(1-\frac{1}{4}\frac{\Omega^2}{\omega_L^2}\right)^2.
\end{aligned} \tag{32}$$

Figure 1 shows Γ_+ and $\Gamma_{+\mathrm{RWA}}$ as functions of the Rabi driving Ω. $\Gamma_{+\mathrm{RWA}}$ is a constant for $0 < \Omega < \omega_L$ but Γ_+ decreases with increasing Ω. At $\Omega = \omega_L$, $\Gamma_+/\Gamma_{+\mathrm{RWA}} = 4/9$. This is to say that the effect of counter-rotating terms is to reduce the decay rate of the driven two-level system coupled with its dissipative environment. In other words, the counter-rotating terms lead to the interference between the driving and dissipation in the spin-boson model which makes the decay rate become weaker.

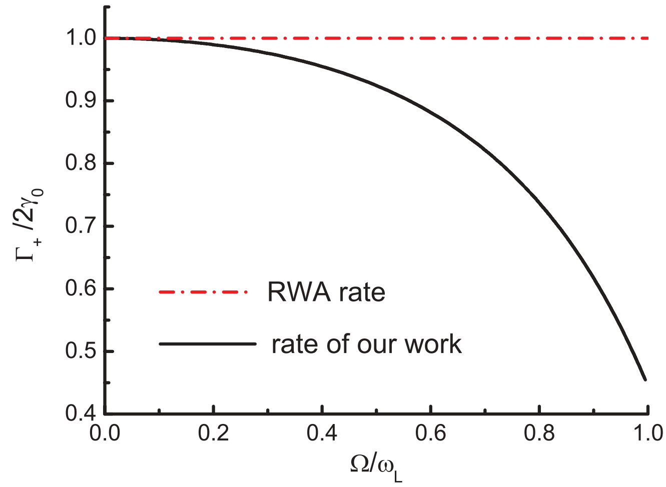


FIG. 1. (Color online) Γ_+ and $\Gamma_{+\mathrm{RWA}}$ as functions of the Rabi driving Ω.

Note that $\Gamma_{\mathrm{RWA}} = [\gamma_0+\Gamma_{+\mathrm{RWA}}]/2 = 3\gamma_0/2$ and $\Gamma_{+\mathrm{RWA}}/2 = \gamma_0$ make the decay rate of $\tilde{\rho}_{21}(t)$ and $\tilde{\rho}_{12}(t)$ [Eqs. (27) and (28)] the same as what Mollow obtained [6] from the RWA. But in Mollow's work [6] $\Gamma_- = 0$ and thus,

$$\begin{aligned}
P_M(t) &= \cos(\omega_L t)e^{-\gamma_0 t} + \left[-\frac{2\gamma_0}{\Omega}+\frac{2\gamma_0}{\Omega}\cos(\Omega t)e^{-\frac{3}{2}\gamma_0 t}\right.\\
&\quad \left. + \frac{3\gamma_0^2}{\Omega^2}\sin(\Omega t)e^{-\frac{3}{2}\gamma_0 t}\right]\sin(\omega_L t),
\end{aligned} \tag{33}$$

where the subscript M means the Mollow's solution for $\Omega \gg \gamma_0$.

For the zero-driving case, $\Omega = 0$, $P(t) = \cos(\omega_L t)\exp(-\gamma_0 t)$ is a simple damped oscillation for both the RWA and our non-RWA treatment, since we have $\gamma(\eta\Delta) = \gamma_0(\Delta) = \gamma_0$. For finite driving our calculated $P(t)$ is plotted as a function of dimensionless time $\omega_L t$ for $\alpha = 0.01$ in Fig. 2 ($\Omega = 0.25\omega_L$), Fig. 3 ($\Omega = 0.5\omega_L$), and Fig. 4 ($\Omega = 0.75\omega_L$), where the dashed-dotted line is the RWA result. Compared with the RWA result, our result predicts a longer coherent time and

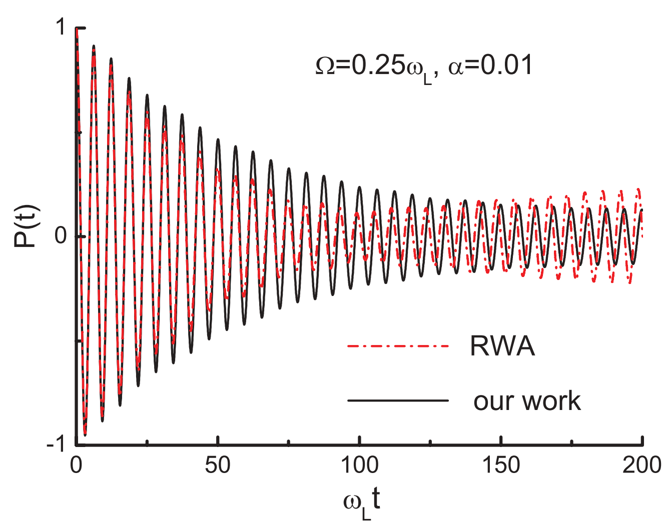
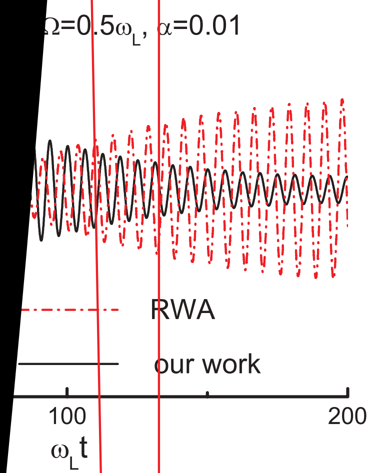


FIG. 2. (Color online) $P(t)$ as a function of dimensionless time $\omega_L t$ for $\alpha = 0.01$ and $\Omega = 0.25\omega_L$.

[redacted]



[redacted] t) as a function of dimensionless time [redacted] $5\omega_L$.

[redacted] is to say that a nonzero driving plays [redacted] quantum dynamics of the two-level [redacted] mic bath.

[redacted] r decay rate, Figs. 2–4 also show that [redacted]

[redacted] online) The phase shift ϕ as a function of the Rabi [redacted] driving Ω/ω for dissipation effect of the environment because $\gamma(\omega)$, as a function of frequency ω, is proportional to the spectral density $J(\omega)$ [Eqs. (24) and (25)] and $\gamma(\eta\Delta \pm \Omega)$ means that the decay rates can be controlled by changing the Rabi frequency Ω. But in the diagonalization approach used in [17] we can get $\gamma(\eta\Delta)$ only as the long-time decay rate because, as we said in Sec. I, the Rabi driving by $\Omega\cos(\omega_L t)$ between the excited states $|s_1\rangle|\{1_k\}\rangle$ and $|s_2\rangle|\{1_k\}\rangle$ has not been taken into account.

In this work, we consider the Ohmic bath with spectral function $J(\omega) = 2\alpha\omega\theta(\omega_c - \omega)$ because it may be the most interesting one [1,2]. Our approach can be used for other structureless spectral functions, such as the super-Ohmic and sub-Ohmic baths, as well as the structured bath with Lorentz-type spectral function. For the structured bath, because there are some characteristic frequencies related to the structure, the interference between the driving and the dissipation may be more interesting than the structureless bath. The related work is underway.

ACKNOWLEDGMENTS

This work was supported by the National Natural Science Foundation of China (Grant No. 10734020) and the National Basic Research Program of China (Grant No. 2011CB922202).

APPENDIX: MASTER EQUATION AND BORN-MARKOV APPROXIMATION

In the interaction picture, the density operator is

$$\tilde{\rho}^I_{SB}(t) = \exp(i\tilde{H}_0 t)\tilde{\rho}_{SB}(t)\exp(-i\tilde{H}_0 t), \qquad \text{(A1)}$$

which satisfies the equation,

$$\frac{d}{dt}\tilde{\rho}^I_{SB}(t) = -i\big[H_I(t), \tilde{\rho}^I_{SB}(t)\big]. \qquad \text{(A2)}$$

Here, $H_I(t)$ is the perturbation H_1' in the interaction picture,

$$H_I(t) = \exp(i\tilde{H}_{0S}t)H_1'(t)\exp(-i\tilde{H}_{0S}t),$$

$$H_1'(t) = \sum_k V_k[b_k^\dagger|s_1\rangle\langle s_2|e^{i(\omega_k-\eta\Delta)t} + b_k|s_2\rangle\langle s_1|e^{-i(\omega_k-\eta\Delta)t}]. \qquad \text{(A3)}$$

Equation (A2) can be solved by iteration,

$$\frac{d}{dt}\tilde{\rho}^I_{SB}(t) = -i\big[H_I(t), \tilde{\rho}^I_{SB}(0)\big] - \int_0^t \big\{H_I(t), \big[H_I(t'), \tilde{\rho}^I_{SB}(t')\big]\big\}dt'. \qquad \text{(A4)}$$

The Born approximation is to approximately decouple (A4) by assuming $\tilde{\rho}^I_{SB}(t') \approx \tilde{\rho}^I_S(t')\rho_B$ in the integration, that is, all higher order (than g_k^2) terms are neglected. The reduced density operator is $\tilde{\rho}^I_S(t) = \mathrm{Tr}_B\tilde{\rho}^I_{SB}(t)$ and the master equation for reduced density operator is

$$\frac{d}{dt}\tilde{\rho}^I_S(t) = -\int_0^t \mathrm{Tr}_B\big[H_I(t), [H_I(t'), \tilde{\rho}^I_S(t')\rho_B]\big]dt', \qquad \text{(A5)}$$

since $\mathrm{Tr}_B[H_I(t), \tilde{\rho}^I_{SB}(0)] = 0$. Returning back to the Schroedinger picture,

$$\tilde{\rho}^I_S(t) = \exp(i\tilde{H}_{0S}t)\tilde{\rho}_S(t)\exp(-i\tilde{H}_{0S}t) \qquad \text{(A6)}$$

with $\tilde{H}_{0S} = \Omega(|s_2\rangle\langle s_1| + |s_1\rangle\langle s_2|)/2$, the master equation becomes

$$\frac{d}{dt}\tilde{\rho}_S(t) = -i[\tilde{H}_{0S}, \tilde{\rho}_S(t)] - \int_0^t dt'\mathrm{Tr}_B\big[H_1'(t), e^{i\tilde{H}_{0S}(t'-t)}[H_1'(t'), \tilde{\rho}_S(t')\rho_B]e^{-i\tilde{H}_{0S}(t'-t)}\big]. \qquad \text{(A7)}$$

$H_1'(t)$ is in Eq. (A3). The Markov approximation is assuming $\tilde{\rho}_S(t') \approx \tilde{\rho}_S(t)$ in the integration, which makes the integral-differential equation become a differential equation.

In this paper we consider the zero temperature case and the trace operation Tr_B can be done easily,

$$\begin{aligned}\frac{d}{dt}\tilde{\rho}_S(t) = {}& -i[\tilde{H}_{0S}, \tilde{\rho}_S(t)] - \int_0^t dt' \sum_k V_k^2 \\ &\times\big\{e^{-i(\omega_k-\eta\Delta)(t-t')}|s_2\rangle\langle s_1|e^{i\tilde{H}_{0S}(t'-t)}|s_1\rangle\langle s_2|\tilde{\rho}_S(t)e^{-i\tilde{H}_{0S}(t'-t)} \\ &- e^{i(\omega_k-\eta\Delta)(t-t')}|s_1\rangle\langle s_2|e^{i\tilde{H}_{0S}(t'-t)}\tilde{\rho}_S(t)(|s_2\rangle\langle s_1|e^{-i\tilde{H}_{0S}(t'-t)} \\ &- e^{-i(\omega_k-\eta\Delta)(t-t')}e^{i\tilde{H}_{0S}(t'-t)}|s_1\rangle\langle s_2|\tilde{\rho}_S(t)e^{-i\tilde{H}_{0S}(t'-t)}|s_2\rangle\langle s_1| \\ &+ e^{i(\omega_k-\eta\Delta)(t-t')}e^{i\tilde{H}_{0S}(t'-t)}\tilde{\rho}_S(t)|s_2\rangle\langle s_1|e^{-i\tilde{H}_{0S}(t'-t)}|s_1\rangle\langle s_2|\big\}.\end{aligned} \qquad \text{(A8)}$$

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