Quantum Zeno and Anti-Zeno Effects: Without the Rotating-Wave Approximation

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We show that the counterrotating, neglected in the previous studies of the quantum Zeno effect (QZE) in atomic decay, can have a large impact on the short-time evolution. We calculate the electron self-energy, the Lamb shift, and the QZE without making the rotating-wave approximation (RWA) and show that, for hydrogen in free space, the Zeno time is longer by 2 orders of magnitude than that obtained from the RWA. We also show that there is no anti-Zeno effect as the counterrotating terms and rotating terms represent the opposite processes in the higher frequency region. Consequently, the experimental measurement of the QZE may be much easier than what was determined with the RWA results.

It is well known that the counterrotating terms are important in obtaining the correct Lamb shift. However, their contribution is insignificant in the calculation of the decay rate for the excited state of an atom in the long time limit. In this Letter, we address the following question: What is the effect of the counterrotating terms on the dynamical evolution of the atom at short times, particularly on the quantum Zeno and anti-Zeno effects? We find the surprising result that the effect of the counterrotating terms can lead to a longer Zeno time by 2 orders of magnitude and that there is no anti-Zeno effect.

The quantum Zeno effect (QZE) and anti-Zeno effects (AQZE) have been widely discussed theoretically [1–6] and experimentally [7]. In the experiment of Itano *et al.* [7], the QZE was observed on the coherent population transition from lower to the upper levels via frequent measurements, and the time interval is of the order of *ms*. In the experiment of Fischer *et al.* [7], the QZE and AQZE of frequent measurements on the escaped numbers of atoms tunnelling away from a trap were presented, and the reported time interval was about 1 μs. For the decay from an excited level of an atom interacting with an environment, frequent measurements at an extremely short-time interval slow down the decay process (the QZE) because the decay of the excited state is almost zero at the beginning of the decay process [1,2]. It is also found that if the measurement time interval is short, but not extremely short, the decay of the excited state could be accelerated [2] (the AQZE). Let $P(\tau)$ denote the survival probability (after a short-time interval τ) at the initial state, which can be written as $P(\tau) = \exp[-\gamma(\tau)\tau]$. After N time measurements at equal τ ($N\tau = t$), the survival probability of the excited level reads $P(t) = P^N(\tau) = \exp[-\gamma(\tau)N\tau] = \exp[-\gamma(\tau)t]$ with $\gamma(\tau)$ the effective decay rate. If $N = 1$, $P(t) = \exp[-\gamma(t)t]$, which goes to $P(t) \to \exp(-\gamma_0 t)$ for large enough t, where γ_0 is the decay rate under the Weisskopf-Wigner approximation. We will have the QZE if $\gamma(\tau) < \gamma_0$, and the AQZE if

$\gamma(\tau) > \gamma_0$. However, the experimental investigation of QZE and AQZE on the decay of a real atom has not been reported so far due to the extremely short measurement time interval predicted from previous theoretical studies ($<10^{-15}$ s).

It is well known that the whole spectrum of the environment (not only the part around the atomic transition frequency) is important for the QZE and AQZE. In the previous studies on QZE and AQZE [1–6], usually the two-level model with the rotating-wave approximation (RWA) is used. An interesting problem relates to a real multilevel atom with the counterrotating terms that are neglected in RWA.

In this Letter, we study the effect of the counterrotating terms on QZE and AQZE in the decay of a real multilevel atom without the RWA (the decay from $2P$ to $1S$ of the hydrogen atom as a concrete example). We show that the neglected terms in RWA can have the same order contribution as the spectral components off-resonant with the atomic transition frequency. This is particularly important for the QZE where the measurement time interval is extremely short [1,2,4].

Usually, the environment is described by some quantum field, which may be in an infinite space [4] or in a confined box [2,5]. The high frequency region of the spectrum of these two cases is different. In infinite space, we may have the ultraviolet (UV) divergence, so we need a renormalizable theory. The interaction for a real multilevel atom interacting with the electromagnetic (EM) field in free space can be described by the Hamiltonian $H = H_0 + H_1$ [8,9] (setting $\hbar = 1$),

$$H_0 = \sum_i E_i |i\rangle\langle i| + \sum_k \omega_k b_k^\dagger b_k, \qquad (1)$$

$$H_1 = \sum_i \sum_{j\neq i} \sum_k g_{k,ij}(b_k^\dagger + b_k)|i\rangle\langle j|, \qquad (2)$$

where summation $\sum_i$ is for all levels $i = 0, 1, 2, \ldots$ and $\sum_{j\neq i}$ for all levels j except $j = i$. $b_k^\dagger$ (b_k) is the creation

(annihilation) operator of EM mode with frequency ω_k (k including the polarization), and $g_{k,ij} = -e\sqrt{1/2\epsilon_0\omega_k V}\mathbf{e}_k \cdot \mathbf{p}_{ij}/m$ is the coupling between the atom and the EM field, with $\mathbf{e}_k$ the polarization vector and $\mathbf{p}_{ij}$ the transition matrix element of momentum operator between the levels i and j. Usually, the coupling can be characterized by the interacting spectrum [2,4,9]: $G_{ij}(\omega) = \sum_k g_{k,ij}^2 \delta(\omega - \omega_k)$.

The Hamiltonian H cannot be solved exactly, and usually the RWA is used [2,4,8], which leads to the following results. Starting from the initial state $|\psi(0)\rangle = |I\rangle|\{0_k\}\rangle$ (the atom in level I, $|I\rangle$, and the EM field in the vacuum, $|\{0_k\}\rangle$), the survival amplitude of finding the system still in $|\psi(0)\rangle$ at $\tau > 0$ is [2,4]

$$x_{\mathrm{RWA}}(\tau) = \frac{1}{2\pi i}\int_B \frac{e^{p\tau}dp}{p + iE_I + \sum_k \sum_{j<I}\frac{g_{k,Ij}^2}{p + i\omega_k + iE_j}}, \quad (3)$$

where B is the so-called Bromwich path and the summation $\sum_{j<I}$ is for all levels j with $E_j < E_I$. Here, RWA is to drop all counterrotating terms with factor $\exp[\pm i(\omega_k + |E_i - E_j|)]$. The survival probability in the initial state is $P_{\mathrm{RWA}}(\tau) = |x_{\mathrm{RWA}}(\tau)|^2 = \exp[-\gamma_{\mathrm{RWA}}(\tau)\tau]$ and the effective decay rate $\gamma_{\mathrm{RWA}}(\tau)$ for a short time τ is given by [2],

$$\gamma_{\mathrm{RWA}}(\tau) = 2\pi\int_0^\infty d\omega \sum_{j<I} G_{Ij}(\omega)F(\omega - \omega_{Ij}, \tau), \quad (4)$$

$$F(\omega - \omega_{Ij}, \tau) = 2\sin^2\left[\frac{\omega - \omega_{Ij}}{2}\tau\right]\Big/\pi\tau(\omega - \omega_{Ij})^2, \quad (5)$$

where $\omega_{Ij} = E_I - E_j$. Since $F(\omega - \omega_{Ij}, \tau) \to \delta(\omega - \omega_{Ij})$ for large enough τ, we have the decay rate $\gamma_0 = 2\pi\sum_{j<I}G_{Ij}(\omega_{Ij})$ in the Weisskopf-Wigner approximation.

When the counterrotating terms are included, the above method is no longer valid. Here, we present an analytical approach, based on unitary transformation and perturbation theory to calculate the survival amplitude and the effective decay rate for Hamiltonian (1) in order to clarify the impact of the counterrotating terms on the short-time evolution and on the QZE and AQZE.

First, we take a unitary transform [10] on H, $H' = \exp(iS)H\exp(-iS)$, with

$$S = \sum_i \sum_{j\neq i} \sum_k \frac{g_{k,ij}\xi_{k,ij}}{i\omega_k}(b_k^\dagger - b_k)|i\rangle\langle j|, \quad (6)$$

where $\xi_{k,ij}$ is a k-dependent function. The transform can be done order by order, $H' = H_0 + H_1' + H_2' + O(g_k^3)$, where $O(g_k^3)$ contains terms of order g_k^3 and higher, and will be neglected. By choosing the following functional form for $\xi_{k,ij}$,

$$\xi_{k,ij} = \frac{\omega_k}{\omega_k + |E_j - E_i|}, \quad (7)$$

the first order terms (of order g_k), $H_1' = H_1 + [iS, H_0]$, become

$$H_1' = \sum_k \sum_{j>i} \frac{2g_{k,ij}\xi_{k,ij}}{\omega_k}|E_j - E_i|(|i\rangle\langle j|b_k^\dagger + |j\rangle\langle i|b_k). \quad (8)$$

Note that H_1' is of the same form as in the RWA coupling. The second order terms are $H_2' = [iS, H_1] + \frac{1}{2}\times\{iS, [iS, H_0]\}$,

$$H_2' = -\sum_i \sum_{j\neq i} \sum_k \frac{g_{k,ij}^2}{\omega_k}\left\{2\xi_{k,ij} - \xi_{k,ij}^2 - \xi_{k,ij}^2\frac{E_j - E_i}{\omega_k}\right\}|i\rangle\langle i|$$
$$+ V_{\mathrm{nd}}, \quad (9)$$

where V_{nd} contains the nondiagonal terms, $|i\rangle\langle j|$ ($i \neq j$) for the atom and $b_k^\dagger b_{k'}^\dagger$, $b_k b_{k'}$, $b_k^\dagger b_{k'}$, and $b_k b_{k'}^\dagger$ ($k \neq k'$) for the EM field. Since the contribution of these nondiagonal terms to the physical quantities is of the fourth order in g_k and can be neglected, we will drop V_{nd} in the following calculation.

The summation $\sum_k$ can be replaced by the integral [9]

$$\sum_k \frac{g_{k,ij}^2}{\omega_k}h(\omega_k) = \frac{2\alpha}{3\pi(mc)^2}\mathbf{p}_{ij}^2\int_0^\infty d\omega_k h(\omega_k), \quad (10)$$

where $h(\omega_k)$ is any function of ω_k and α is the fine structure constant. Then, it can be easily seen that the first term in H_2' is linear divergent in the UV limit. The divergence comes from the self-energy of the free electron due to the vacuum fluctuations, $E_{\mathrm{se}} = -\sum_k \sum_{j\neq i}[g_{k,ij}^2/\omega_k]|i\rangle\langle i| = -\sum_{j\neq i}\frac{2\alpha\omega_c}{3\pi(mc)^2}\mathbf{p}_{ij}^2|i\rangle\langle i|$ ($\omega_c \approx mc^2$ is the UV cutoff), which does not depend on the atomic level structure. The divergence can be removed by the mass renormalization with a subtraction of the self-energy E_{se} [9],

$$H_2' - E_{\mathrm{se}} = -\sum_i \sum_{j\neq i} \sum_k \frac{g_{k,ij}^2}{\omega_k}$$
$$\times\left\{2\xi_{k,ij} - \xi_{k,ij}^2 - 1 - \xi_{k,ij}^2\frac{E_j - E_i}{\omega_k}\right\}|i\rangle\langle i|$$
$$= \frac{2\alpha}{3\pi(mc)^2}\sum_i\left\{\sum_{j<i}\mathbf{p}_{ij}^2\omega_{ij}\right.$$
$$\left. + \sum_{j\neq i}\mathbf{p}_{ij}^2(E_j - E_i)\ln\frac{\omega_c + |\omega_{ij}|}{|\omega_{ij}|}\right\}|i\rangle\langle i|. \quad (11)$$

The transformed Hamiltonian can be divided into unperturbed part and a perturbed part as $H' \approx H_0' + H_1'$. Here, $H_0' = H_0 + H_2' - E_{\mathrm{se}}$ is the unperturbed part,

$$H_0' = \sum_i E_i'|i\rangle\langle i| + \sum_k \omega_k b_k^\dagger b_k, \quad (12)$$

$$E_i' = E_i + \frac{2\alpha}{3\pi(mc)^2}\sum_{j<i}\mathbf{p}_{ij}^2\omega_{ij}$$
$$+ \frac{2\alpha}{3\pi(mc)^2}\sum_{j\neq i}\mathbf{p}_{ij}^2(E_j - E_i)\ln\frac{\omega_c + |\omega_{ij}|}{|\omega_{ij}|}, \quad (13)$$

and H_1' in Eq. (8) represents the perturbation.

The ground state of $H' \approx H_0' + H_1'$ is $|0\rangle|\{0\}_k\rangle$, where $|0\rangle$ is the lowest state of the atom. The initial state for the QZE is the excited state $|I\rangle|\{0\}_k\rangle$. Then, the survival amplitude of finding the system in the initial state is $x(\tau) = \langle\{0_k\}|\langle I|\exp(-iH'\tau)|I\rangle|\{0_k\}\rangle$. Since $H' \approx H_0' + H_1'$ is of

the form of the RWA Hamiltonian, $x(\tau)$ can be calculated in the same way as above for $x_{\mathrm{RWA}}(\tau)$ in Eq. (3),

$$x(\tau) = \frac{1}{2\pi i} \int_B \frac{e^{p\tau} dp}{p + iE_I' + \sum_k \sum_{j<I} \frac{V_{k,Ij}^2}{p + i\omega_k + iE_j'}}, \quad (14)$$

where $V_{k,Ij} = 2\omega_{Ij} g_{k,Ij} \xi_{k,Ij}/\omega_k$. The Lamb shift of the level I can be obtained from the imaginary part of the pole of integrand in (14),

$$E_I^O = E_I' + \sum_k \sum_{j<I} \frac{V_{k,Ij}^2}{E_I' - \omega_k - E_j'}$$
$$= E_I + \frac{2\alpha}{3\pi(mc)^2} \sum_{j\neq I} \mathbf{p}_{Ij}^2 (E_j - E_I) \ln \frac{\omega_c + |\omega_{Ij}|}{|\omega_{Ij}|}. \quad (15)$$

The second term is the Lamb shift of level I, which is exactly the same as obtained by the previous authors [9]. The superscript O for E_I^O means the observed energy of level I.

The survival probability in the initial state is $P(\tau) = |x(\tau)|^2 = \exp[-\gamma(\tau)\tau]$, and the effective decay rate $\gamma(\tau)$ for a short time τ is

$$\gamma(\tau) = 2\pi \int_0^\infty d\omega \sum_{j<I} G_{Ij}'(\omega) F(\omega - \omega_{Ij}, \tau), \quad (16)$$

where $G_{Ij}'(\omega) = 4G_{Ij}(\omega)(\omega_{Ij})^2/(\omega + \omega_{Ij})^2 = G_{Ij}(\omega) \times f_{Ij}(\omega)$ and $G_{Ij}(\omega) = \frac{2\alpha\omega}{3\pi(mc)^2} \mathbf{p}_{Ij}^2$.

The spectrum is modulated by the function $f_{Ij}(\omega) = [1 - (\omega - \omega_{Ij})/(\omega + \omega_{Ij})]^2$, where the second term is due to the counterrotating terms which is proportional to $1/(\omega + \omega_{Ij})$ and is zero at $\omega = \omega_{Ij}$ because the decay rate at large enough time does not depend on the counterrotating terms. In Fig. 1, we plot $G_{Ij}(\omega)/G_{Ij}(\omega_{Ij})$ and $G_{Ij}'(\omega)/G_{Ij}(\omega_{Ij})$ versus ω/ω_{Ij}. It is interesting to note that both curves are independent of the prefactor $\frac{2\alpha}{3\pi(mc)^2} \mathbf{p}_{Ij}^2$ of $G_{Ij}(\omega)$. For the modification function $f_{Ij}(\omega)$, we have $f_{Ij}(\omega) < 1$ for $\omega > \omega_{Ij}$, and $f_{Ij}(\omega) \ll 1$ for $\omega \gg \omega_{Ij}$. This means that the counterrotating terms greatly suppress the higher frequency part of the interacting spectrum. This results in an important effect on the short-time behavior of the system.

The above discussion is suitable for any multilevel unstable quantum system. For numerical calculation, we consider the $2P - 1S$ transition of the hydrogen atom [4]. Let $I = 2P$ and $j = 1S$ in Eqs. (3), (4), (14), and (16), and the interacting spectrum is [4] $G(\omega) = \chi\omega/[1 + (\omega/\omega_c)^2]^4$, where χ is a constant. The transition frequency is $\omega_0 = E_{2P} - E_{1S}$. In Fig. 2, $\gamma(\tau)/\gamma_0$ is plotted for $\omega_0 = E_{2P} - E_{1S} = 1.55 \times 10^{16}$ rad/s and $\omega_c = 0.85 \times 10^{19}$ rad/s [4]. The dashed line is the result of RWA, and we see that, for an extremely short time ($\tau\omega_0 < 1.5 \times 10^{-4}$), RWA predicts the QZE and for a short time ($\tau\omega_0 > 1.5 \times 10^{-4}$) as well as the AQZE (the largest value of $\gamma_{\mathrm{RWA}}(\tau)/\gamma_0 \approx 38$). However, by taking into account the
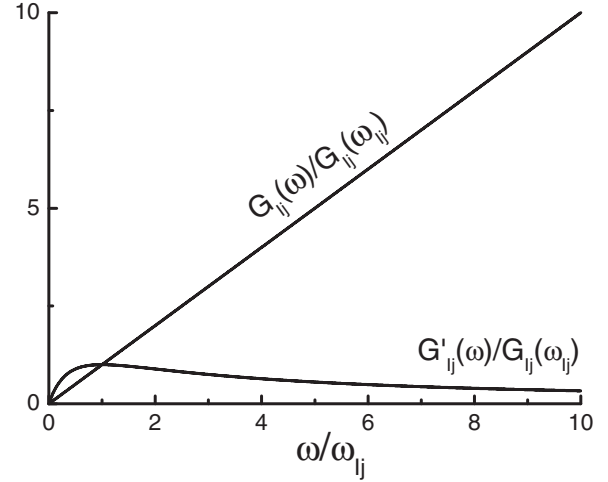

FIG. 1. $G_{Ij}(\omega)$ (dotted line) and $G_{Ij}'(\omega)$ (solid line) versus ω.

counterrotating terms, we only have the QZE and no AQZE, in contradiction to the universality claim in [2].

The different behavior of $\gamma(\tau)$ and $\gamma_{\mathrm{RWA}}(\tau)$ can be understood by checking $G(\omega)$ and $G'(\omega)$ in Eqs. (4) and (16). The modification function $f(\omega)$ due to the counterrotating terms on the spectrum $G(\omega)$ is $f(\omega) < 1$ for $\omega > \omega_0$. When ω_0 is much smaller than the spectrum maximum frequency $\omega_{\max}$, the spectrum $G'(\omega)$ is greatly suppressed compared with $G(\omega)$, see Fig. 1. The dephasing function [2] $F(\omega - \omega_0, \tau)$ is mainly a single-peak function with peak at ω_0 and width $\sim 1/\tau$. Since the integrand in Eq. (4) is $G(\omega)F(\omega - \omega_0, \tau)$, when ω_0 is far below the maximum of $G(\omega)$, we find that $\gamma_{\mathrm{RWA}}(\tau)$ grows with decreasing τ (AQZE) because $F(\omega - \omega_0, \tau)$ is then probing more of the rising part of $G(\omega)$. Our result is different from Ref. [2] because the integrand in Eq. (16) is $G'(\omega)F(\omega - \omega_0, \tau)$ and $\gamma(\tau)$ decreases with decreasing τ (QZE) since $F(\omega - \omega_0, \tau)$ already covers the main part of $G'(\omega)$. The physics will be discussed below.

For the short-time limit, the survival probability is quadratic in τ: $P(\tau) = 1 - \tau^2/\tau_Z^2$ for $\tau \to 0$, which is explic-
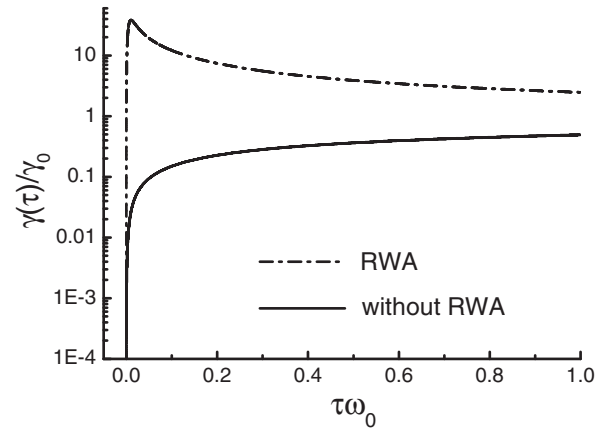

FIG. 2. Decay rate $\gamma(\tau)$ for $\omega_0/\omega_c = 0.001824$. The dashed line is the result of RWA.

itly different from the exponential decay [2–5]. The "Zeno time"[3–5], τ_Z, defines the time for the Zeno effect and can be calculated by using Eq. (16),

$$\tau_Z = \left(\frac{d}{d\tau}\gamma(\tau)\right)^{-1/2}\bigg|_{\tau\to 0} = \left(\int_0^\infty d\omega \sum_{j<I} G'_{Ij}(\omega)\right)^{-1/2},$$
(17)

while the Zeno time in the RWA is $\tau_Z^{\mathrm{RWA}} = [\int_0^\infty d\omega \sum_{j<I} G_{Ij}(\omega)]^{-1/2}$. By using the spectrum for hydrogen atom [4], we obtain $\tau_Z^{\mathrm{RWA}} = \sqrt{6/\chi}\,\omega_c^{-1}$ and (for $\omega_c/\omega_0 \gg 1$)

$$\tau_Z = \frac{1}{2\omega_0\sqrt{\chi}}\left(\ln\frac{\omega_c}{\omega_0} - \frac{23}{12}\right)^{-1/2}.$$
(18)

In Fig. 3, we plot τ_Z versus ω_0. Note that τ_Z is dependent on the transition frequency ω_0 due to the counterrotating terms, while τ_Z^{RWA} is independent of ω_0 as the integrand $G(\omega)$ does not depend on ω_0. The dependence of τ_Z on ω_0 is physically correct, since the short-time evolution $P(\tau) = 1 - \tau^2/\tau_Z^2$ should depend on where the transition frequency ω_0 is located in the interacting spectrum. Compared with τ_Z^{RWA}, the counterrotating terms lead to a longer Zeno time, especially when the atomic transition frequency is very small. For hydrogen atom, $\chi = 6.4 \times 10^{-9}$, and we have $\tau_Z^{\mathrm{RWA}} = 3.6 \times 10^{-15}$ s [4] but $\tau_Z = 1.9 \times 10^{-13}$ s. The latter is nearly 2 orders of magnitude larger than the former. Therefore, the real experimental measurement of QZE is much easier than what was determined with the RWA results. Under current technology, a measurement time interval of 10^{-13} s is achievable.

The physics can be easily understood. Consider the two terms (one from the counterrotating and one from rotating terms) in the original Hamiltonian in the interaction picture, $b_k^\dagger e^{i\omega_k t}(|1S\rangle\langle 2P|e^{-i\omega_0 t} + |2P\rangle\langle 1S|e^{i\omega_0 t})$. For $\omega_k \gg \omega_0$, the two terms are almost the same, but represent opposite atomic processes, decay from the excited to ground levels, and jumping from ground to the excited levels. They cancel each other, which is consistent with $f_{2P,1S}(\omega) = [1 - (\omega - \omega_0)/(\omega + \omega_0)]^2 \to 0$ for $\omega/\omega_0 \gg 1$. Therefore, the counterrotating terms make the decay much slower comparing with the decay with only the rotating-wave terms in this case (in the higher frequency region).

In summary: We have shown that, besides the Lamb shift and electron self-energy, the counterrotating terms have great impact on the short-time evolution of the population of the excited level, and thus on the quantum Zeno and anti-Zeno effects (Our calculation is limited to those QZE and AQZE where the rotating-wave approximation is used). For the short-time evolution, the counterrotating terms can be included, simply by multiplying the RWA spectrum $G(\omega)$ with the factor $[1 - (\omega - \omega_0)/(\omega + \omega_0)]^2$. We present the analytical study for the multilevel atom coupled to the EM field, based on the unitary transformation and perturbation method. With this method, we
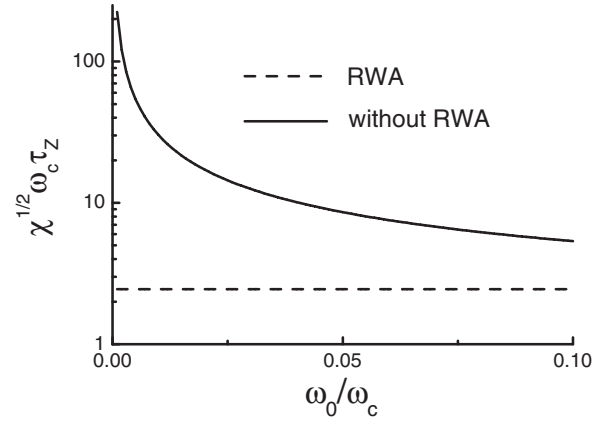

FIG. 3.   The Zeno time as a function of the transition frequency ω_0.

can simultaneously obtain the electron self-energy, Lamb shift, and the short-time evolution (the QZE and AQZE). The Zeno time depends on the atomic transition frequency sensitively due to the counterrotating terms. For the hydrogen atom, we calculate the effective decay rate and find that, because of the counterrotating terms, the Zeno time is much longer than the Zeno time obtained with RWA, and there is no anti-Zeno effect.

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