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Position-dependent dynamics of two qubits in a leakage cavity

Jun Jing[1,3], Zhi-Guo Lü[2], Hong-Ru Ma[2] and Hang Zheng[2]

[1] Department of Physics, Shanghai University, Shanghai 200444, People's Republic of China
[2] Department of Physics, Shanghai Jiao Tong University, Shanghai 200240, People's Republic of China
[3] The Shanghai Key Lab of Astrophysics, Shanghai, People's Republic of China

E-mail: jungen@shu.edu.cn and zglv@sjtu.edu.cn

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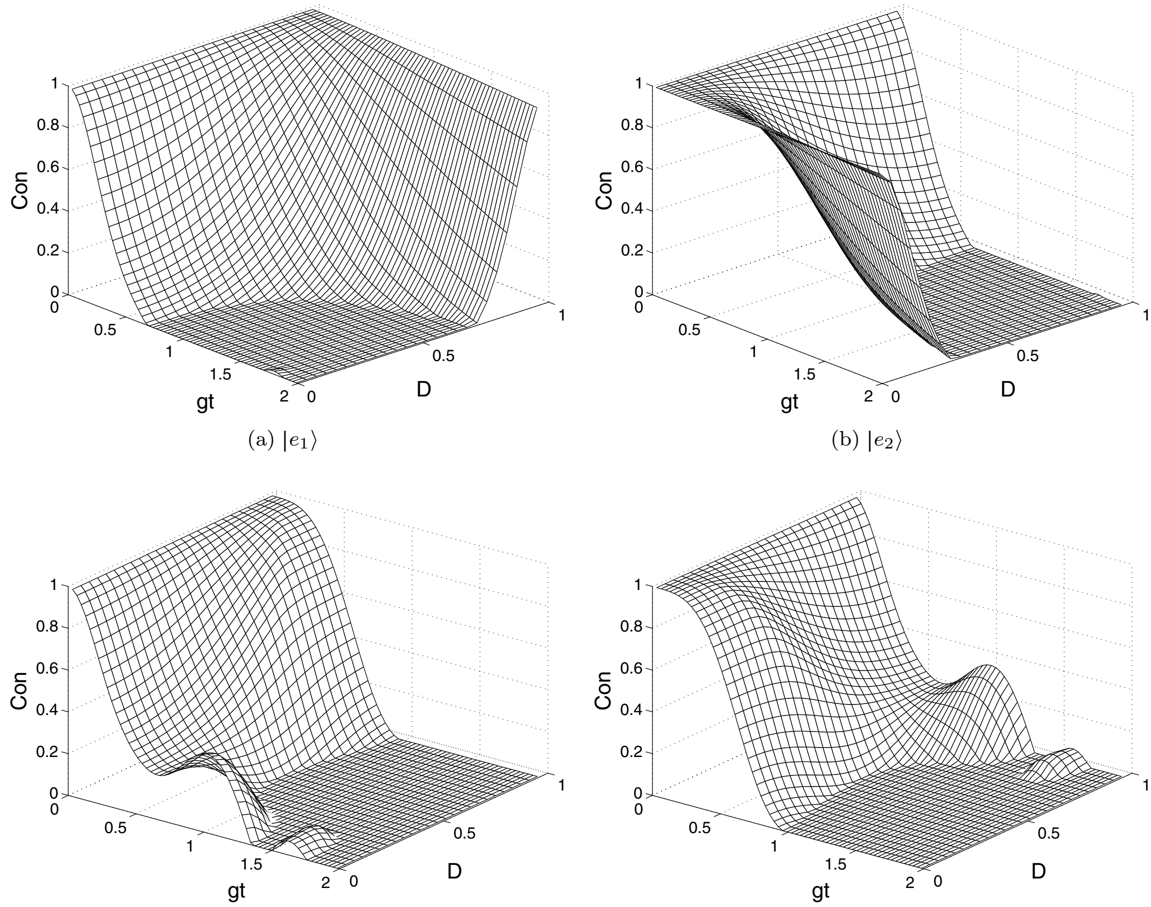
Abstract

The position-dependent entanglement dynamics of two qubits embedded in a leakage cavity is investigated. The two qubits are initialled in Bell states and the cavity mode is taken as a standing wave. It is found that (i) the dynamics of the Bell states can be divided into two groups according to one-photon entangled states and two-photon entangled states; (ii) the entanglement life of the one-photon entangled states can be kept as long as possible if we put the two qubits at certain positions; (iii) at larger detuning, the entanglement dynamics manifests more robustly.

1. Introduction

Quantum dissipation including decoherence and disentanglement represents an important quantum statistical mechanical problem, whose diversity ranges from quantum–classical transition [1] to limitations of quantum information [2]. One of the prototype models in this field is the Caldeira–Leggett model which describes a quantum particle in a dissipative bath of harmonic oscillators [3]. It is an idea of an open quantum system based on a spin–boson model, which means that any system should be thought of as being surrounded by its environment (reservoir or bath) which influences its dynamics. The spin–boson model provides a natural approach for discussing dissipation process (damping and dephasing), where the centre system consisting of two-level atoms or pseudo-spins-1/2 is interesting because it displays both a localized (classical) and a delocalized (quantum) phase for the effect of the boson field modes [4, 5]. Models of this kind were intensively investigated, in which a fundamental one named the Jaynes–Cummings (JC) model [6] set a very important milestone in the early days of quantum optics. With the development of theoretical and experimental techniques, especially in the field of cavity quantum electrodynamics (CQED) [7–9], the JC model and its extensions (with more atoms and/or more boson modes as well as with or without rotation wave approximation) have been exploited to understand quantum decoherence, i.e., the vanishing of the off-diagonal elements of the spin-reduced density matrix in any basis and the dynamics of entanglement between the centre spins as well as that between the spin and the environmental boson, i.e., the whole system results in such a state that cannot be factorized [10] in its Hilbert space.

Recently, a lot of works based on those CQED systems (JC-like) were devoted to implementing quantum communications [11–14] or engineering entanglement between atoms in optical cavities. It is not only a fundamental issue in the theory of quantum mechanics, but also is involved with creating, quantifying, controlling, distributing and manipulating the entangled quantum bits [2, 15–17]. Many interesting phenomena and their explanations or applications are being discovered. For instance, generally, the entanglement degree between the centre spins vanishes asymptotically due to different kinds of quantum reservoirs. However, if the reservoir consists of, e.g., only one or two electromagnetic field modes, then the entanglement may decrease abruptly and non-smoothly to zero in a finite time [13, 18, 19], which is a new nonlocal decoherence called entanglement sudden death (ESD). Natali and Ficek discuss the spontaneous emission effects in entanglement creation in an optical cavity [20]. However, some effects, such as the leakage of the cavity, the temperature and the noise arising from the interaction between the atom and the environment modes have actually not been considered thoroughly. The cavity boundary condition in former works can be neglected, since the size of the atoms is much less than the wavelength of the optical light. Yet currently, the size of solid quantum



(a) $|e_1\rangle$ (b) $|e_2\rangle$ (c) $|e_3\rangle$ (d) $|e_4\rangle$

Figure 1. Time evolution of the concurrence for two qubits initialled in Bell states at different positions with $\omega_0 = 2.0g$, $\omega = g$, $\gamma = 0.1$ and $n = 1$.

dots, a good candidate for quantum information process, can be comparable with the optical wavelength. Then the boundary condition and the configuration of qubits should be reconsidered seriously. Therefore, the demonstration of the dynamics of centre qubits in different realizations of QCED models is still an open and non-trivial question.

In this paper, we consider two qubits or pseudo-spins of $1/2$ (as an open subsystem with qubits labelled 1 and 2) in a leaking single-mode cavity. The photon losses due to imperfect reflectivity of the cavity mirrors should be taken into account in the realistic description of this CQED [21] system. And the two qubits initialled as one of the four Bell states are located at different positions in the cavity with the standing wave condition. Although we only place them in a symmetric configuration and their state parameters such as energy biases and the coupling strengths with the boson mode are identical, they are still in asymmetry phases which depend on their positions. In the present work, we focus on the short-time dynamical effect arising from the positions of the atoms and cavity leakage. This is related to the scenarios for realizing quantum computation via Raman interaction of quantum dots embedded in a micro-cavity, such as the atomic CQED system [22–24] or the quantum-dot CQED [25]. The initial most-entangled states will help us to understand the non-unitary and degrading quantum reduced evolution, which is measured by the concurrence [26, 27] between the two centre qubits. The rest of this paper is organized as follows. In section 2 the model Hamiltonian is introduced. Detailed results and discussions are in section 3. Finally, we present our conclusions in section 4.

2. Model and Hamiltonian

Generally, a model in the CQED with leakage can be represented by a Lindblad master equation:

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} = -\mathrm{i}[H, \rho] + \gamma \left(a\rho a^\dagger - \frac{1}{2}a^\dagger a\rho - \frac{1}{2}\rho a^\dagger a \right), \qquad (1)$$

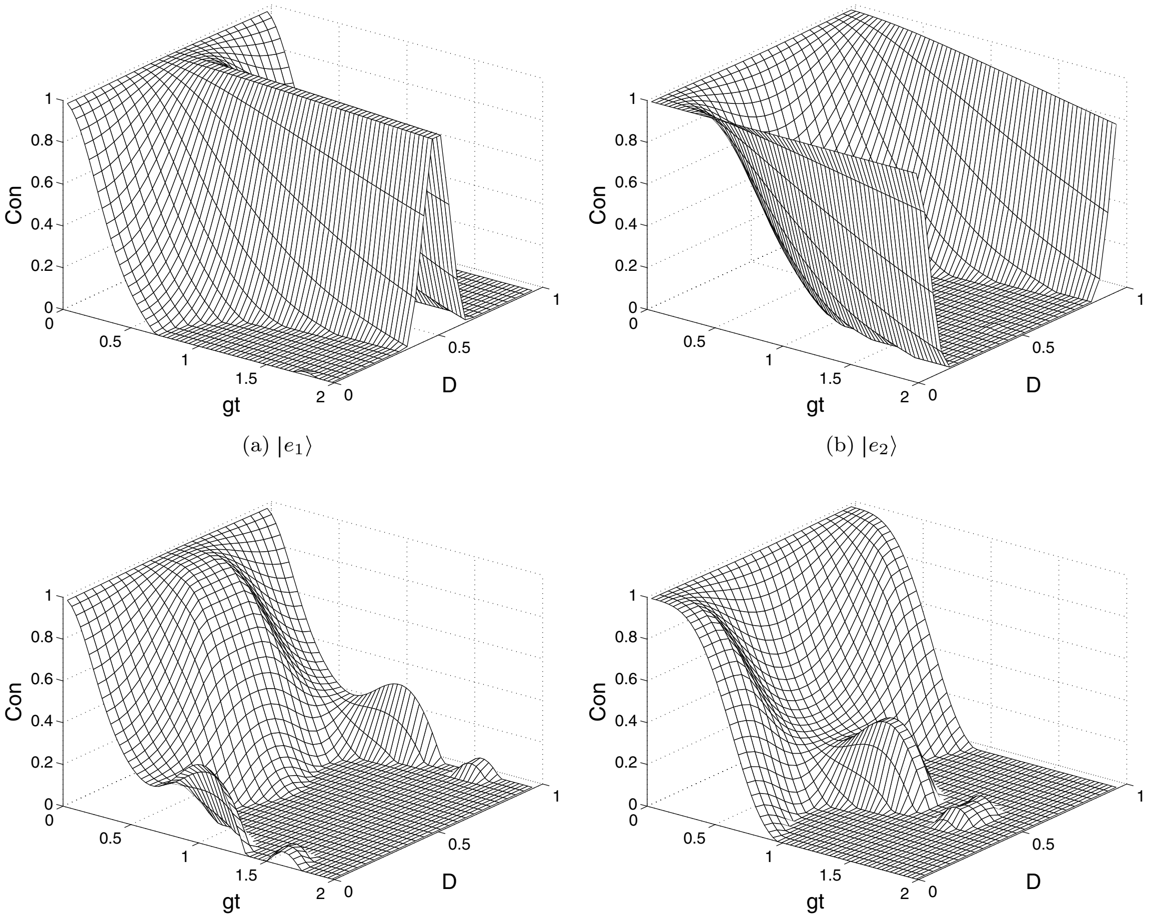
where ρ is the density matrix of the atom–cavity system. γ describes the rate of the photons leaking out of the cavity. The Hamiltonian which determines the unitary dynamics of the whole system is

$$H = H_0 + H_I, \qquad (2)$$

$$H_0 = \frac{\omega_0}{2}\left(\sigma_z^1 + \sigma_z^2\right) + \omega a^\dagger a, \qquad (3)$$

$$H_I = g\sigma_x^1(a^\dagger \mathrm{e}^{-\mathrm{i}qr_1} + a\mathrm{e}^{\mathrm{i}qr_1}) + g\sigma_x^2(a^\dagger \mathrm{e}^{-\mathrm{i}qr_2} + a\mathrm{e}^{\mathrm{i}qr_2}). \qquad (4)$$

The energy bias between the two states is given by ω_0 ($\hbar = 1$). ω is the photon energy and g is the coupling strength between



(a) $|e_1\rangle$ (b) $|e_2\rangle$

(c) $|e_3\rangle$ (d) $|e_4\rangle$

Figure 2. Time evolution of the concurrence for two qubits initialled in Bell states at different positions with $\omega_0 = 2.0g$, $\omega = g$, $\gamma = 0.1$ and $n = 2$.

the photon and the qubits. $\sigma_i (i = x, y, z)$ is the Pauli matrix. The positions of the two qubits are $r_{1,2} \in (0, L)$, where L is the size of the cavity and set as 1.0 for simplicity in the following. For the boundary condition of a standing wave, $qL = n\pi, n = 1, 2, 3, \ldots$. The two qubits are placed in a symmetry configuration: $r_1 + r_2 = 1.0, r_1 \in (0, 0.5], r_2 \in [0.5, 1.0)$. Then we define $D = r_2 - r_1$ as the distance between them. Obviously $D \in [0, 1)$. When $D = 0$, qubit-1 and qubit-2 are in the same position (the middle of the cavity), which means the interaction part of the Hamiltonian 2 is reduced to a position-independent JC-like model.

3. Numerical simulation results and discussions

At the beginning, the total system is assumed to be separable, i.e., $\rho(0) = |\psi(0)\rangle\langle\psi(0)| \otimes \rho_b$. The centre subsystem $|\psi(0)\rangle$ is prepared as one of the Bell states,

$$|e_1\rangle = 1/\sqrt{2}(|10\rangle + |01\rangle),$$
$$|e_2\rangle = 1/\sqrt{2}(|10\rangle - |01\rangle),$$
$$|e_3\rangle = 1/\sqrt{2}(|11\rangle + |00\rangle),$$
$$|e_4\rangle = 1/\sqrt{2}(|11\rangle - |00\rangle),$$

while the bath (the single mode in the cavity) is in its vacuum state $|0\rangle$. After exact numerical calculation, we can determine $\rho(t)$ from $\rho(0)$ by equation (1). Tracing out the degrees of freedom of the environment, we finally obtain the reduced matrix of the two qubits: $\rho_S(t) = Tr_b(\rho(t))$. We discuss the intra-entanglement dynamics by concurrence, which is defined by:

$$C = \max\{\lambda_1 - \lambda_2 - \lambda_3 - \lambda_3, 0\} \qquad (5)$$

and $\lambda_i, i = 1, 2, 3, 4$, are the square roots of the eigenvalues of the product matrix $\rho_S(\sigma_y \otimes \sigma_y)\rho_S^*(\sigma_y \otimes \sigma_y)$ in decreasing order. The concurrence is a very good entanglement degree measurement for a two two-level atom system and applies to a pure and mixed state.

3.1. *Preparation with $\omega_0 = 2.0\omega$*

Then we calculate the time evolution of the concurrence as a function of D, the distance between the two qubits in the cavity. In figure 1, we set $n = 1$ for the standing wave condition, where the centre-symmetrical atoms are in-phase. In figure 2, we set $n = 2$, where the atoms in the two corresponding positions are out-of-phase. In both figures, we choose $\omega_0 = 2.0\omega$ for a moderate detuning case and in the following subsection, we discuss a large detuning one. The fact that the

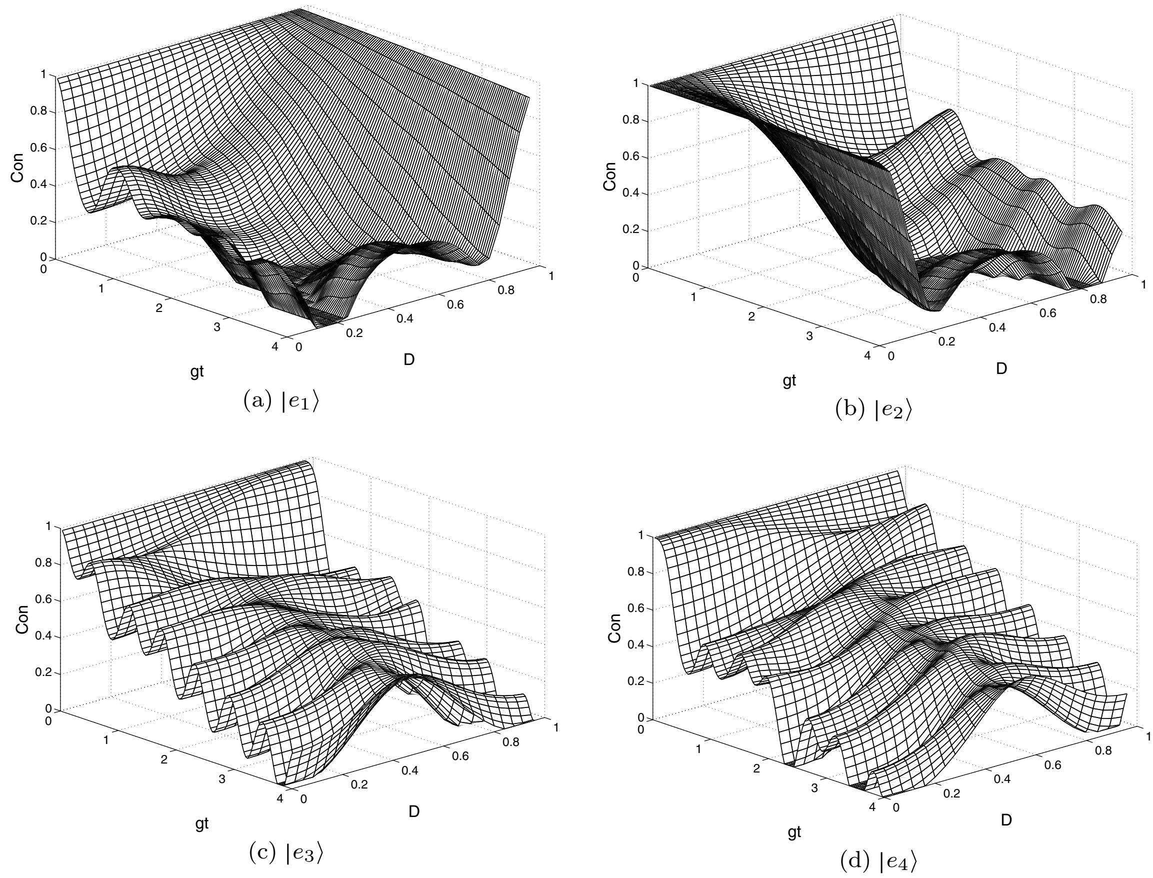


Figure 3. Time evolution of the concurrence for two qubits initialled in Bell states at different positions with $\omega_0 = 5.0g$, $\omega = g$, $\gamma = 0.1$ and $n = 1$.

leaking rate of photons $\gamma \neq 0$ determines the destiny of the concurrence of the Bell states. Yet the calculation results can help us to understand the differences among the four important most-entangled states and the position-dependent dynamics.

In figure 1(a), the sudden death time, during which the concurrence decreases to the value of zero abruptly, is increased by changing D from 0 towards 1. This means when the two qubits are separated from each other step by step, the entanglement of the initial Bell state $|e_1\rangle = 1/\sqrt{2}(|10\rangle + |01\rangle)$ can survive longer and longer. However, there is a reverse tendency of the state $|e_2\rangle = 1/\sqrt{2}(|10\rangle - |01\rangle)$ with increasing D. In another word, in figure 1(b), it is found that when $D = 0$, the concurrence practically remains at its maximum value 1.0. As known, $|e_2\rangle$ is a decoherence-free state when $\gamma = 0$. Similarly, the position-dependent effects on $|e_3\rangle = 1/\sqrt{2}(|11\rangle + |00\rangle)$ and $|e_4\rangle = 1/\sqrt{2}(|11\rangle - |00\rangle)$ are also completely opposite. However, in figures 1(c) and (d), the variation of distance between the two qubits is not helpful to prolong the entanglement sudden death time obviously. Yet it is worth noting that in the condition of shorter distance for the state $|e_3\rangle$, or in the condition of longer distance for the state $|e_4\rangle$, the concurrence has an obvious revival process during the interval $gt \in (1.5, 2.0)$.

If $n = 2$, the middle point of the cavity is a wave node instead of an antinode in the case of $n = 1$. Therefore, the phase in r_1 and r_2 is opposite and the entanglement dynamics behaves totally differently from that in the case of $n = 1$. For the state $|e_1\rangle$ in figure 2(a), when D approaches 0.5, the concurrence seems to enjoy a fairly long life. It is easy to find that $D = 0.5$ means the two qubits just stand at the two antinodes inside the cavity. While in figure 2(b), the tendency of $|e_2\rangle$ is contrary to that of $|e_1\rangle$. The concurrence quickly damps unless D moves towards 0 or 1. The two extreme values suggest that the qubits are at wave nodes. During the short time interval we are interested in, both of the two states have no revival phenomena after their entanglement becomes 'dead'. In the positions where $|e_2\rangle$ obtains a long life, $|e_3\rangle$ has two revivals during the short-time evolution (see figure 2(c)). And if the qubits are near to two different antinodes respectively, $|e_4\rangle$ damps with an obvious entanglement revival.

3.2. Preparation with $\omega_0 = 5.0\omega$

In the condition of a larger detuning (here we set $\omega_0 = 5.0\omega$), the concurrence dynamics is qualitatively consistent with the above results. Comparing figure 3 with figure 1, it is roughly that the farther the distance between the two qubits, the longer the entanglement life for the centre system. The entanglement is more robust for large detunings rather than small or even zero detuning. It is partly due to the fact that in condition of a larger detuning, the effective coupling ($g = 1/5\omega_0$) between the qubit and the cavity is weaker than that of the smaller one

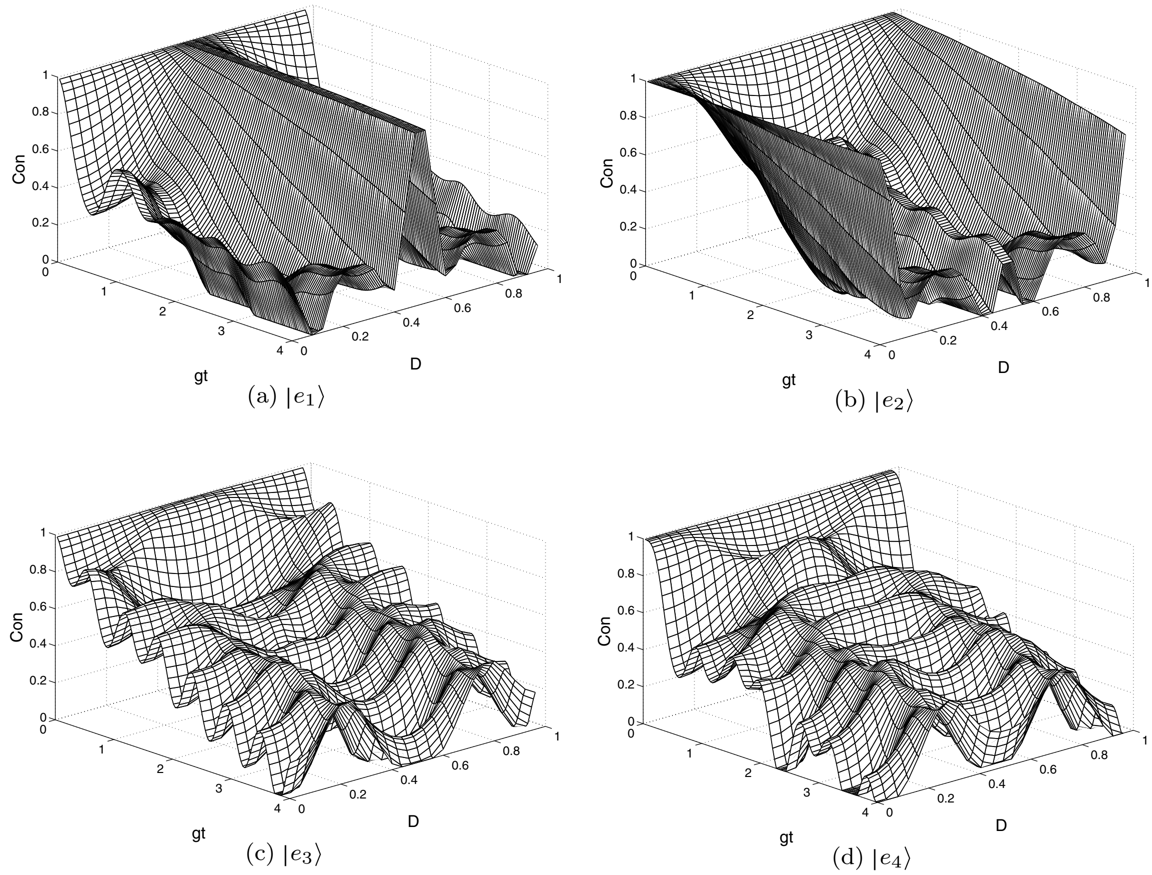


Figure 4. Time evolution of the concurrence for two qubits initialled in Bell states at different positions with $\omega_0 = 5.0g$, $\omega = g$, $\gamma = 0.1$ and $n = 2$.

($g = 1/2\omega_0$). Thus, in the off-resonance case (large detuning), the loss of entanglement from the bell state is slower than that of near-resonance (small detuning).

In figure 3(a), after the moment $gt = 2.0$, the entanglement degree is still above 0.2 whatever the distance D is. Around $D = 0.4$, the dynamics shows a more complex process. There occurs an entanglement sudden death (about $gt = 2.0$) and a concurrence revival (about $gt = 3.0$). The whole pattern of figure 3(b) is also reverse to that of figure 3(a) as well as the relationship between figures 1(b) and (a). In figures 3(c) and (d), the concurrence damps with a long-term vibration; and ESD tends to occur when D approaches 0 (both qubits are at the antinode) and 1 (they are at the node). And the high detuning can help to maintain the entangled qubits if their distance is around $D = 0.5$.

Although the spontaneous emission of the qubits is ignored in the condition of moderate detuning (to see subsection 3.1), the pattern of figure 2 is qualitatively the same as that in a large detuning condition (to see figure 4). Yet the latter shows more complex details, which corresponds to the dynamics of the $n = 1$ case in figure 3. It is not difficult to find that one half (from $D = 0$ to $D = 0.5$) of the four sub-figures in figure 4 ($n = 2$) is consistent with the corresponding ones in figure 3 ($n = 1$).

In summary, Bell states can be divided into two groups, $|e_{1,2}\rangle$ (one-photon entangled states or anti-parallel states) and $|e_{3,4}\rangle$ (two-photon entangled states or parallel states). The former group has opportunities to keep its entanglement degree as long as possible if we place them at certain positions: in case of $n = 1$, the two qubits initialled in $|e_1\rangle$ ($|e_2\rangle$) should be at the wave nodes (antinode); and in case of $n = 2$, $|e_1\rangle$ ($|e_2\rangle$) should be at the antinodes (wavenodes). The results of $n > 2$ are analogous to the above two cases. And it is well known, if the entanglement degree between qubits can be robust against the dissipation effect from their environment, such as the leakage from the cavity, it is very useful to the applications of the quantum dot CQED system, which is the basis of the realization of gate operations of quantum computation. For the latter one, their dynamics are also dependent on the qubits' positions. It is justified to omit the spontaneous emission only in the case of a large detuning. However, our calculation shows that most basic features can be discovered in any nonzero detuning case, such as the one we choose.

4. Conclusion

In this paper, two qubits initialled as Bell states are placed in symmetrical positions in the centre of a leaking cavity. The cavity is in the condition of a standing wave. In two different standing wave cases, we calculate the time evolution of the four Bell states under moderate and large detuning conditions. All of the results show that the entanglement dynamics is a sensitive function of the position of the qubits inside the cavity and the initial states. This implies that only by changing the positions of the qubits in the cavity (or inversely the size of the cavity in practice), we can control their dynamics. And when the qubits are near to the antinodes or nodes, their dynamics can be utilized in the process of quantum information.

Many works still remain to continue the present one, such as: (i) what if the qubits are not placed at positions symmetrical to the centre of the cavity? (ii) When we move them simultaneously from the centre position towards different edges of the cavity, could this approach contribute to the entanglement revival? These issues are under our consideration.

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